MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 106 No. 6

## MEETING OF 1946 NOVEMBER 8

Professor H. H. Plaskett, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Robert Malcolm Coates, Master Mariner, A.I.N.A., c/o National Provincial Bank Ltd., Cornmarket Street, Oxford;

Ralph William Nicholls, A.R.C.S., Demonstrator in Physics, Imperial College of Science and Technology, S. Kensington, London, S.W. 7;

Merle Tuberg, Yerkes Observatory, Williams Bay, Wis., U.S.A.; and National Research Fellow in Theoretical Astrophysics, Girton College, Cambridge; and

George Alfred Tunnell, B.Sc., Meteorological Office, Harrow; and 75 St. Joseph's Drive, Southall, Middlesex.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Charles Gordon Abel, 8 Tankridge Road, Cricklewood, London, N.W. 2 (proposed by H. Wildey);

Angus Armitage, M.Sc., Lecturer in the History and Philosophy of Science in the University of London, University College, Gower Street, London, W.C. 1; and 52 Parkhill Road, London, N.W. 3 (proposed by H. Dingle);

Hugh Townshend Vesey Church, F.R.Met.Soc., Redroof, Felpham, Bognor Regis, Sussex (proposed by H. Hancock);

Kenneth Roy Coleman, B.Sc., 78 Sheaf Lane, Sheldon, Birmingham 26 (proposed by M. C. Johnson);

Hubert Roland Cooper, Master Mariner, Post Office, Saldanha, Cape Province, S. Africa (proposed by R. H. Stoy);

Cyril Tetlow Daltry, B.Sc., Tutor in Mathematics, University of London Institute of Education, Malet Street, London, W.C. 1; and 99 Maze Hill, Greenwich, London, S.E. 10 (proposed by L. M. Milne-Thomson);

William Henry Douglas Heavens, Boars Hill, Oxford (proposed by Vera Reade);

Albert Francis Arthur Lofley Jones, 40 Trafalgar Street, Timaru, New Zealand (proposed by D. C. Berry);

Sir Philip Magnus, Bart., M.A., The Athenaeum Club, Pall Mall, London, S.W. 1 (proposed by L. M. Milne-Thomson);

Graham James Odgers, Solar Physics Observatory, Cambridge (proposed by F. J. M. Stratton);

Alan Bernard Partridge, B.A., Peterhouse, Cambridge (proposed by F. J. M. Stratton);

Edward H. Preston, Hayden Planetarium, American Museum of Natural History, Central Park West at 79th Street, New York 24, U.S.A. (proposed by Clyde Fisher);

William Ernest Reynolds, Brockley, Netherfield Drive, Guiseley, near Leeds, Yorkshire (proposed by J. F. Cox);

Herbert Edgar Sweet, F.R.G.S., F.R.Met.Soc., Brix, 21 Rutland Road, Maidenhead, Berkshire (proposed by J. C. W. Herschel);

Frederick Edward Townend, M.B.E., 39 Park Avenue, Grimsby, Lincolnshire (proposed by W. M. F. Macfarlane);

Brian Gilbert Wales-Smith, 1 Mount Park Crescent, Ealing, London, W. 5 (proposed by G. W. Wakeford); and

George Gilbert Yates, M.A., Solar Physics Observatory, Cambridge (proposed by F. J. M. Stratton).

Thirty-seven presents were announced as having been received since the last ordinary meeting.

## MEETING OF 1946 DECEMBER 13

Professor H. H. Plaskett, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Roland Lebeg Townley Clarkson, The Bungalow, The Avenue, Trimley St. Mary, Ipswich, Suffolk;

Arthur William Lane Hall, Devon Lodge, Hartington Road, Sherwood, Nottingham;

Marjorie Hall Harrison, Yerkes Observatory, Williams Bay, Wis., U.S.A.; and

Guy Ernest Williamson, M.B.E., Hildegarde, Wells Road, Bickley, Kent.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Arthur Francis O'Donel Alexander, M.A., Ph.D., Cartref, 1 Athelstan Road, Dorchester (proposed by M. A. Ellison);

David Robert Bates, M.Sc., Lecturer in Mathematics in the University of London, University College, Gower Street, London, W.C. 1 (proposed by H. S. W. Massey);

Pedro Marcos Bilbao, 10 The Grange, Maitland Park Road, London, N.W. 3 (proposed by P. M. Ryves);

Sydney Goochild, 1st Bn. Sikh Light Infantry, Poona, India (proposed by E. A. Glennie);

Neville James Goodman, Fosswyn, Luton Road, Markyate, Hertfordshire (proposed by J. P. M. Prentice);

Henry Howard Hammond, M.B.E., 26 Southdean Gardens, Wimbledon Park, London, S.W. 19 (proposed by F. M. Holborn);

Maurice Neville Hill, B.A., King's College, Cambridge (proposed by E. C. Bullard);

Kurt August Hirsch, Ph.D., D.Phil., 22 Northcote Road, Leicester (proposed by S. Goldstein);

Horace Charles Colin Hunt, 5 Stoke Road, Aylesbury, Buckinghamshire (proposed by F. M. Holborn);

Harry Ernest Ruddy, M.A., Braunston Rectory, Rugby, Warwickshire (proposed by E. L. Hawke);

John Herbert Jennings Westlake, The Cedars, Victoria Road, Deal, Kent (proposed by H. Wildey); and

Patrick Lever Willmore, B.A., St. John's College, Cambridge (proposed by E. C. Bullard).

Sixty presents were announced as having been received since the last meeting, including:—

*Nouvelle Biographie Universelle* (presented by Miss M. F. Hardcastle);

*Manual of the Admiralty Gyro-compass* (*Sperry Type*) (presented by Mr C. T. Spencer); and

*Historical Records of the Survey of India*, Vol. I—18th Century (presented by the Geodetic Branch of the Survey of India).

Mr H. F. Finch and Dr V. C. A. Ferraro were appointed honorary auditors of the Treasurer's accounts for the year 1946.

# SOME ASPECTS OF STELLAR EVOLUTION—I

## Direct Condensation

*K. E. Edgeworth*

(Received 1946 October 10)

*Summary*

It is postulated that the stars have been formed from a rotating cloud of gas, dust and small solid particles, in which gas predominates, and the hypothesis that the stars have been formed by a process of direct condensation is examined.

The theory of gravitational instability fixes a minimum distance between condensations, and it is shown that this rule involves the conclusion that the stars must have been formed at a temperature very little above the absolute zero, a result which appears to be highly improbable. This is the first difficulty. It is aggravated if it is assumed that an appreciable proportion of the stellar mass is due to accretion.

The local angular momentum of a spherical volume of material forming part of a rotating disk such as the Galaxy is of the order $a^2\omega$ or $a^2V/r$ and not $a^2dV/dr$. Applying this formula to the case of the solar system, the angular momentum is found to be much larger than the actual angular momentum of the system as a whole and very much larger than the angular momentum of the Sun alone. This is a further difficulty.

On account of these difficulties the hypothesis that the stars have been formed by a process of direct condensation would appear to be untenable.

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*Introduction.*—It is very generally accepted that the material which now forms the stars, the planets and other members of the Galactic system was originally distributed more or less uniformly in space, and that it has since condensed under the influence of gravitation to form the system that we are able to observe at the present day. The precise character of the evolutionary process is by no means clear, however, and there are certain theoretical difficulties to be met before it can be claimed that we possess even a superficial knowledge of the course of events.

At the outset it may be remarked that the problem is essentially one which should be viewed as a whole. Theories of the Galaxy or of the solar system may be plausible in themselves but may be inconsistent with one another. Or again, difficulties that have arisen in trying to explain certain features of the solar system may disappear when viewed against the correct stellar background.

The motive power which actuates the whole mechanism of stellar evolution is the force of gravitation, and the operation of this force is controlled and restricted by two other factors—rotation and gas pressure. The course of events is determined by the interplay of these three factors.

There are three types of evolutionary process that appear to be deserving of consideration:—

(i) Direct condensation in the original cloud of gas.

(ii) Successive condensations involving the formation of intermediate structures of disk-like form, smaller than the Galaxy, but larger than the stars.

(iii) The formation of rings and filaments which break up in due course to form the stars.

The present paper deals with the first of these three possibilities.

*Rotating disk.*—At the present time the Galaxy is in the form of a rotating disk, and it may be assumed that the shape of the primitive cloud from which the stars were formed was somewhat similar. In a rotating disk of gas there are certain relationships between thickness, density and temperature which will be required in the course of our analysis.

Consider a column of gas at right angles to the central plane.

The following notation will be used:—

$z$ = height of any point above the central plane;
$p$ = pressure at height $z$; $p_0$ = pressure at $z=0$;
$\rho$ = density at height $z$; $\rho_0$ = density at $z=0$;
$c$ = molecular velocity of the gas in the central plane;
$T$ = temperature; $\mu$ = molecular weight of the gas;
$\mathfrak{R}$ = gas constant.

Although the disk has no definite boundary surfaces, it is convenient to postulate boundary surfaces which include about 98 per cent. of the total mass.

Let $t$ = thickness of the disk, that is to say the distance between the two boundaries.
$\bar{\rho}$ = mean density.
$\sigma$ = mass of the disk per unit area.

Let us postulate that $\bar{\rho}=\frac{1}{2}\rho_0$, so that

$$\sigma = t\bar{\rho} = \tfrac{1}{2}t\rho_0. \tag{1}$$

Assuming that the heat generated is free to escape, we have $p=k\rho$, where $k=\mathfrak{R}T/\mu$.

The gravitational attraction of the disk at any point outside the boundary is $2\pi G\sigma$.

Inside the boundary the force of gravity may be assumed to be

$$2\pi G\sigma z/\tfrac{1}{2}t = 4\pi\bar{\rho}Gz, \tag{2}$$

so that

$$dp/dz = -4\pi\rho\bar{\rho}Gz, \tag{3}$$

$$kd\rho/dz = -4\pi\rho\bar{\rho}Gz, \tag{4}$$

$$\rho = \rho_0 e^{-b^2z^2}, \tag{5}$$

where $b^2=2\pi\bar{\rho}G/k$.

Now

$$\sigma = \int_{-\infty}^{\infty}\rho dz = \rho_0\int_{-\infty}^{\infty}e^{-b^2z^2}dz = 1{\cdot}8\rho_0/b; \tag{6}$$

therefore

$$b = 1{\cdot}8\rho_0/\sigma = 3{\cdot}6t^{-1} \tag{7}$$

and

$$k = \frac{2\pi\rho G}{b^2} = \frac{2\pi}{13}\cdot\frac{G\sigma^2}{\bar{\rho}}. \tag{8}$$

But $k = \mathfrak{R}T/\mu$ and $\sigma = t\bar{\rho}$, so that

$$t = \frac{13}{2\pi}\frac{\mathfrak{R}T}{\mu G\sigma}, \quad \text{say}^*, \quad \frac{2\mathfrak{R}T}{\mu G\sigma}. \tag{9}$$

Putting $\mathfrak{R} = 8{\cdot}3 \times 10^7$; $G = 6{\cdot}7 \times 10^{-8}$; $\mu = 1$, we get

$$t = 2{\cdot}5 \times 10^{15} T/\sigma, \tag{10}$$

$$\bar{\rho} = \sigma/t = 4 \times 10^{-16}\sigma^2/T, \tag{11}$$

$$c^2 = 2{\cdot}5\mathfrak{R}T/\mu = 2{\cdot}1 \times 10^8 T = 1{\cdot}25G\sigma t = 1{\cdot}25G\bar{\rho}t^2. \tag{12}$$

*Numerical values.*—Since numerical calculations are involved in our discussion, it is necessary to use formulae expressed in terms of such quantities as can be determined with fair accuracy by observation.

Let $R$ = distance of the Sun from the centre of the Galaxy $= 3 \times 10^{22}$ cm.

$M$ = mass of the Galaxy inside the orbit of the Sun $= 3 \times 10^{44}$ gm.

$\omega$ = angular velocity of the Sun in its orbit.

Now $R^3\omega^2 = GM$, therefore $\omega = 9 \times 10^{-16}$.

Let $\bar{\sigma} = M/\pi R^2 = 10^{-1}$.

Since the Sun is in the outer regions of the Galaxy, $\sigma$ is presumably less than $\bar{\sigma}$ and it would be reasonable to estimate that its value is about $5 \times 10^{-2}$.

The value of $\sigma$ may also be deduced in another way. Jeans † estimates that the stellar density in the neighbourhood of the Sun is equivalent to one Sun in 800 cubic light years, which works out at $3 \times 10^{-24}$ gm./cm.$^3$. He also estimates that the polar radius of the Galaxy is 5,000 light years, which would mean a thickness of about 6,000 light years or $6 \times 10^{21}$ cm. at the distance of the Sun. This makes $\sigma$ about $2 \times 10^{-2}$ for the stars alone. If an equal amount is added for scattered material in the form of dust $\sigma$ becomes $4 \times 10^{-2}$, which agrees well with the previous estimate.

It may be assumed that $\sigma$ has remained sensibly unchanged during the process of evolution, and the value $\sigma = 5 \times 10^{-2}$ gm./cm.$^2$ will be taken as the value for the primitive material in the neighbourhood of the Sun. Accepting Hoyle's view‡ that the molecular weight is unity, and assuming further that $\sigma$ has the value given above, the molecular velocity, thickness of disk and density can be calculated for any given temperature. These values are given in Table I.

TABLE I

*Rotating disk* ( $\sigma = 5 \times 10^{-2}$)

| $T$ | $c$ | $t$ | $t/2R$ | $\rho$ |
|---|---|---|---|---|
| degrees abs. | cm./sec. | cm. | | gm./cm.$^3$ |
| $10^5$ | $4{\cdot}5 \times 10^6$ | $5 \times 10^{21}$ | 1/12 | $10^{-23}$ |
| $10^4$ | $1{\cdot}4 \times 10^6$ | $5 \times 10^{20}$ | 1/120 | $10^{-22}$ |
| $10^3$ | $4{\cdot}5 \times 10^5$ | $5 \times 10^{19}$ | 1/1,200 | $10^{-21}$ |
| 100 | $1{\cdot}4 \times 10^5$ | $5 \times 10^{18}$ | 1/12,000 | $10^{-20}$ |
| 10 | $4{\cdot}5 \times 10^4$ | $5 \times 10^{17}$ | 1/126,000 | $10^{-19}$ |
| 3 | $2{\cdot}5 \times 10^4$ | $1{\cdot}7 \times 10^{17}$ | 1/400,000 | $3 \times 10^{-18}$ |

Note that $\rho$ varies as $\sigma^2/T$, so that if $\sigma$ is doubled the value of $\rho$ is increased fourfold.

*Gravitational instability.*—In a large cloud of gas gravity creates a tendency for

* F. Hoyle, *M.N.*, **105**, 291, 1945, equation (4) gives a similar formula.

† Sir James Jeans, *The Universe Around Us*, p. 177, 1945.

‡ F. Hoyle, *M.N.*, **105**, 292, 1945.

the cloud to contract upon itself, and the effect of gas pressure is to impose a minimum distance between the embryo condensations.

Denote this minimum distance by $\lambda$.

Then we know that

$$\lambda^2 = \frac{\pi\gamma c^2}{3G\rho}, \quad \text{say,} \quad \frac{c^2}{G\rho}. \tag{13}$$

In applying the formula it is necessary to bear in mind that this value of $\lambda$ is a theoretical minimum, and that condensations of this size would form very slowly. In the case of a rotating disk differences in angular velocity also retard the formation of condensations and must be allowed for.

In the case of a disk we have seen that

$$c^2 = 1{\cdot}25G\bar{\rho}t^2, \quad \text{so that} \quad \lambda^2 = 1{\cdot}25t^2. \tag{14}$$

When the distance between the condensations is greater than the thickness of the disk, the value of $\rho$ in equation (13) is inversely proportional to the radius, so that condensations not much larger than the thickness of the disk are more likely to be formed than larger condensations. Combining these two conditions, we reach the important conclusion that the condensations formed in a rotating disk are likely to be of the same order of magnitude as the thickness of the disk.

Under given conditions there is also a minimum value for the mass of a condensation. The mass of a condensation of radius $r$ ($=\frac{1}{2}\lambda$) is $\pi\sigma r^2$, so that

$$m_{\min} = \tfrac{1}{4}\pi\sigma\lambda^2$$

$$= \sigma t^2 \tag{15}$$

$$= 6 \times 10^{30} T^2/\sigma. \tag{16}$$

Applying these formulae to the Galaxy and putting $\sigma = 5 \times 10^{-2}$, $m = 10^{33}$ we get *

$$T = 2{\cdot}8^\circ \text{ abs.}, \tag{17}$$

which is a temperature so very near the absolute zero that it appears to be very improbable. This is one objection to the hypothesis that the stars have been formed by direct condensation.

*Local angular momentum in a rotating disk.*—The problem of the evolution of the stars from a rotating cloud of scattered material raises the question of the local angular momentum of an embryo condensation.

Let the centre of the disk be taken as origin. Let the point $C(0, r)$ be the centre of an embryo condensation, and let $P(x, r+y)$ be a point close to $C$, that is to say, let $x$ and $y$ be small compared with $r$.

The orbital velocity at $C$ is $V$ and its components are $V$, 0. The orbital velocity at $P$ is $V + ydV/dr$, and its components are $V + ydV/dr$, $xV/r$. The components of velocity of a particle at $P$ referred to the centre of the condensation at $C$ are therefore $ydV/dr$, $xV/r$.

The angular momentum of a particle at $P$ about the centre of the condensation is therefore

$$y^2 dV/dr + x^2 V/r. \tag{18}$$

The actual values of $V/r$ and $dV/dr$ depend upon the distribution of mass.

* F. Hoyle, *M.N.*, **105**, 305, 1945, equation (7) gives $T < 17{\cdot}5(M/M_\odot)^{\frac{1}{3}}$, that is to say $T < 12^\circ$ abs. for $m = 10^{33}$. The difference is largely due to the assumption that $\sigma = 0{\cdot}2$ instead of $0{\cdot}05$.

Let $m$ = the mass of the cloud inside the radius $r$. Then

$$Gm = rV^2$$

and

$$Gdm/dr = V^2 + 2rVdV/dr. \tag{19}$$

But $Gdm/dr$ is necessarily positive, so that

$$\tfrac{1}{2}V/r + dV/dr > 0,$$

$$V/r + dV/dr > \tfrac{1}{2}V/r. \tag{20}$$

If equation (18) is integrated throughout a small spherical volume of radius $A$, the quantities $V/r$ and $dV/dr$ being regarded as constant, the angular momentum per unit mass is of the order

$$A^2(V/r + dV/dr). \tag{21}$$

It is reasonable to say that this quantity, whose minimum value is $\frac{1}{2}A^2V/r$ or $\frac{1}{2}A^2\omega$, is of the order $A^2V/r$. But it is not correct to say, as Hoyle does *, that it is of the order $A^2dV/dr$, since this latter quantity may be zero.

In determining the angular momentum in any particular case there are two possibilities to be considered. If the radius of a condensation is of the same order of magnitude as the thickness of the disk, we may write

$$NA^2 = R^2,$$

where $A$ is the radius of a condensation before contraction takes place, and $N$ is the number of condensations inside a radius $R$.

In that case, writing $\omega'$ for the local angular velocity, the equatorial angular momentum per unit mass is

$$A^2\omega' = R^2\omega'/N. \tag{22}$$

On the other hand, if the radius of a condensation is small compared with the thickness of the disk, we write $t$ for the thickness of the disk, so that

$$\tfrac{4}{3}N\pi A^3 = \pi R^2 t$$

and the angular momentum per unit mass is

$$A^2\omega' = 3R^2t\omega'/4AN, \tag{23}$$

which is higher than before in the proportion $3t/4A$. The value of the angular momentum given by equation (22) is therefore a minimum.

In the case of the Sun we may write $N = 10^{11}$, $R = 3 \times 10^{22}$ and $\omega = 10^{-15}$. Since $\omega'$, the local angular velocity is of the same order as $\omega$, the minimum value of the local angular momentum per unit mass for a star such as the Sun is $10^{19}$.

*Direct condensation.*—Starting with the assumption that the material which now forms the stars and other members of the Galactic system was originally distributed in space in the form of a mixture of gas, dust and small solid particles, in which gas predominated, the first thing that suggests itself is that condensations were formed in the primitive material, and that these condensations contracted upon themselves to form the stars. We are now in a position to review certain aspects of this hypothesis.

In order that the process of condensation should start at all it is necessary that equation (13) should be satisfied, and this at once creates a difficulty.

* F. Hoyle, *loc. cit.*, p. 304.

The question of the temperature of the nebular material is discussed at some length by Hoyle.* He suggests 10,000° abs. as a possible temperature for gas alone and 100° abs. as a possible temperature for a mixture of gas and dust particles, and it must be remembered that it is this temperature which is significant in determining the size of the condensations.

Taking 1000° abs. as a plausible estimate for the temperature, the minimum mass of a condensation is $1{\cdot}2 \times 10^{38}$ gm. or 60,000 times the mass of the Sun. In order to make possible the formation of condensations with a mass of $10^{33}$ gm., it is necessary to postulate a temperature within 2° or 3° of the absolute zero. Unless Hoyle's arguments are entirely fallacious, such a low temperature would appear to be quite inadmissable.

If the suggestion that the stars were originally smaller than they are now and have acquired an appreciable proportion of their masses by accretion is accepted, this difficulty is further accentuated.

The question of angular momentum must also be considered. We have seen that the angular momentum per unit mass of a spherical volume of the galactic cloud having the same mass as the Sun is of the order $10^{19}$. In actual fact the angular momentum per unit mass of the solar system is of the order $1{\cdot}5 \times 10^{17}$ and that of the Sun alone $3 \times 10^{15}$, and the angular momentum of the stars generally would appear to be of similar magnitude. The hypothesis under discussion is therefore open to the further objection that it leads to a value of the angular momentum which is much too high.

These two objections appear to render the hypothesis of direct condensation untenable ; nor is the suggestion of accretion helpful, for, while it diminishes the force of one objection, it increases the force of the other. Some substantial modification of the hypothesis would seem to be required.

*Cherbury,*
*Booterstown,*
*Co. Dublin:*
1946 *October* 8.

* F. Hoyle, *loc. cit.*, pp. 292–298.

# SOME ASPECTS OF STELLAR EVOLUTION—II

## Successive Condensations

*K. E. Edgeworth*

(Received 1946 October 10)

*Summary*

The hypothesis that the stars were formed by a process of successive condensations from a rotating disk is examined. At a temperature of (say) 1000° abs., about a million condensations would be formed, and these might become unstable and break up into smaller condensations of the same mass as the actual stars. The angular momentum of these condensations would be high, and it is possible that they would form binary systems in which the angular momentum would be chiefly associated with the orbital velocity of the components.

This hypothesis appears to meet the difficulties associated with the previous hypothesis of direct condensation which was discussed in Paper I, but it is also open to serious objections. It would lead one to expect that clusters of the order of one hundred thousand stars would be much more numerous than they are, and that close binary systems would be much more numerous than is actually the case. It must be inferred that the hypothesis of successive condensations fails to offer an entirely satisfactory picture of stellar evolution.

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*Introduction.*—In a previous paper it was pointed out that the hypothesis that the stars were formed by a process of direct condensation involves two difficulties. First, the formation of condensations of stellar magnitude would only be possible at a temperature very close to the absolute zero, and second, the angular momentum of the condensations would be far higher than the angular momentum of the solar system.

It would appear that the first difficulty might be met by assuming that the condensations take place in two stages, finally resulting in the formation of rotating clouds of approximately stellar magnitude. To meet the second difficulty it is necessary to postulate a third stage and to assume that a rotating cloud of stellar magnitude follows a different course of evolution from that already described. Instead of breaking up into a large number of small condensations, a cloud of stellar magnitude must be supposed to break up into two components of approximately equal magnitude, thus forming a binary system. The implications of this hypothesis must now be examined.

*The first and second stages.*—It was shown in Paper I that the mass $m$ of a condensation must satisfy the relation

$$m > 6 \times 10^{30} T^2/\sigma.$$

Putting $\sigma = 5 \times 10^{-2}$, $T = 1000°$, we get

$$m > 1{\cdot}2 \times 10^{38},$$

which suggests that the Galaxy might break up into about a million condensations, which would contract upon themselves, forming what may be called sub-galaxies.

It may then be supposed that history repeated itself. The sub-galaxies assumed the form of thin disks and broke up in turn.

For the sub-galaxies $\sigma$ would have the value $5 \times 10^4$ gm./cm.$^2$. With $T = 1000°$ abs. we have

$$m > 3 \times 10^{32},$$

which is a result of the right order. It will thus be seen that the hypothesis of successive condensations appears to remove the first of the difficulties which were involved in the previous hypothesis.

*Angular momentum.*—We use the following notation:—

$N$ = number of stars within a disk of radius $R$;
$n_1$ = number of sub-galaxies formed in this disk;
$n_2$ = number of stellar condensations formed in each sub-galaxy,

so that

$$N = n_1 n_2;$$

$a_1$ = radius of a sub-galaxy after contraction;
$a_2$ = radius of a stellar condensation after contraction;
$\omega$ = angular velocity of the original galactic cloud;
$\omega_1$ = angular velocity of a sub-galaxy;
$\omega_2$ = angular velocity of a stellar condensation.

It will be shown in a later section that

$$a_1 = R/n_1 \quad \text{and} \quad \omega_1 = \omega n_1,$$

so that the angular momentum is (1)

$$R^2\omega/n_1.$$

If a second condensation takes place we have

$$a_2 = a_1/n_2 \quad \text{and} \quad \omega_2 = \omega_1 n_2,$$

so that the equatorial angular momentum per unit mass is

$$a_2^2\omega_2 = a_1^2\omega_1/n_2 = R^2\omega/n_1 n_2 = R^2\omega/N, \tag{2}$$

which is the same as for a direct condensation.

It will thus be seen that the assumption that sub-galaxies are formed as an intermediate stage in stellar evolution is no help in explaining how a condensation gets rid of its superfluous angular momentum. It is for this reason that it is necessary to introduce the additional assumption that there is a third stage in which condensations of stellar magnitude break up again to form binary systems.

*Rotational instability.*—Before proceeding further on the main issue, reference must be made to the theory of rotational instability, which has played an important part in discussions on stellar evolution. The theory is well known and need not be described here. The general idea is that a rotating mass of diminishing volume will pass through a series of dynamically stable shapes, avoiding any form which is dynamically unstable.

The dynamically stable shapes begin with spheroids and pass into ellipsoids. After the ellipsoid no dynamically stable shape is known, but it has been widely supposed that the body will emerge eventually in the form of a binary system in which the bulk of the angular momentum is associated with the orbital velocity. Owing to the fact that no stable shape is known in succession to the ellipsoid, the validity of this assumption has been questioned. In any case, the mathematical

analysis associated with the theory of rotational instability is difficult, and the elegance of the analysis has tended to divert attention from the fundamental assumptions on which the theory is based.

The chief weakness of the theory lies in the assumption that the angular velocity is uniform, an assumption which limits the application of the theory to the special case in which this condition is satisfied. As we shall see in due course, the process of contraction is not likely to be uniform. In the case of a spherical volume of material, originally of uniform density and rotating with uniform angular velocity, contraction is greater near the centre than at the equator and the angular velocity is higher.

The ellipsoidal form is evidently impossible except in the special case in which the angular velocity is uniform, and it is therefore reasonable to suggest that the spheroidal shape must be credited with a greater measure of stability when the density and angular velocity are higher at the centre than at the equator, that is to say, a greater measure of stability than it would possess if the density and angular velocity were uniform.

There is also another point. Any departure from the spheroidal or disk-like shape would involve a redistribution of angular momentum. The only factor which is competent to bring about a change of this nature is viscosity, and viscosity would be very low in the early stages of stellar evolution. It is therefore reasonable to argue that the spheroidal shape possesses a greater measure of stability than has hitherto been assumed, and that polar contraction will in fact continue until the shape of the rotating cloud can be described as a disk.

*The process of condensation.*—In the original Galactic cloud the value of $\sigma$ is of the order $10^{-1}$ gm./cm.$^2$, and it is clear that any heat generated in the process of contraction would escape into space, so that, until the final temperature is reached there would be no development of gas pressure sufficient to cause delay in the process of condensation. In the case of the sub-galaxies the value of $\sigma$ would be of the order $10^5$, and the escape of heat would be slower, but the same argument would be applicable.

In the case of a cloud of stellar mass, on the other hand, the rate at which heat would escape would be so much reduced that the course of events would be substantially modified. It is possible to distinguish three distinct phases:—

(i) A phase of general contraction, which would come to an end when equatorial contraction is arrested by rotation.

(ii) A phase of polar contraction, in which the rotating mass assumes the shape of a disk. This phase comes to an end when the temperature of the cloud reaches a final value.

(iii) The development of gravitational instability, leading to a change of structure.

These three phases must now be examined in somewhat greater detail.

*Contraction*: *First phase.*—Owing to the low density of the Galactic cloud the effects of viscosity would be insignificant, and it will be postulated that the angular momentum of every part of a condensation remains constant during the process of contraction.

In defining the initial form of the condensation two slightly different alternative assumptions are possible. If the diameter of a condensation is of the same order as the thickness of the disk in which it is formed, it may be assumed that the

initial shape of the cloud is cylindrical and that $\sigma$ (the mass per unit area) is constant. If on the other hand the diameter of the condensation is small compared to the thickness of the disk, it may be assumed that the cloud is spherical and that the density $\rho$ is constant.

In either case it is assumed that the local angular velocity is constant and is of the order $\omega$, the angular velocity of the condensation about the centre of the disk.

Let $m$ = mass of the condensation;
$A$ = initial radius of the condensation;
$\omega'$ = initial angular velocity of the condensation;
$a$ = equatorial radius when contraction is halted by rotation;
$\omega_1$ = equatorial angular velocity after contraction.

Then

$$a^2\omega_1 = A^2\omega' \quad \text{and} \quad a^3\omega_1^2 = Gm,$$

so that

$$a = \frac{A^4\omega_1^2}{Gm}, \tag{3}$$

$$\omega_1 = \frac{A^2}{a^2}\omega'. \tag{4}$$

If the size of a condensation is of the same order as the thickness of the disk we may write

$$R^2 = NA^2,$$

$$a = \frac{R^4\omega'^2}{N^2Gm} = \frac{R\omega'^2}{N\omega^2},$$

and if we write $\omega' = \omega$ then

$$a = R/N \quad \text{and} \quad \omega_1 = \omega N. \tag{5}$$

Let $AX$ = distance of any small volume from the centre of the condensation before contraction;
$ax$ = distance of the same material from the centre after contraction;
$m'$ = mass of the material inside the radius $AX$ or $ax$;
$V_x$ = the velocity at distance $ax$;
$\omega_x$ = angular velocity at distance $ax$.

Then

$$a^2x^2\omega_x = A^2X^2\omega' \quad \text{and} \quad a^3x^3\omega_x^2 = Gm',$$

so that

$$x = \frac{A^4X^4\omega'^2}{aGm'} = \frac{m}{m'}X^4, \tag{6}$$

$$\omega_x = \frac{A^2X^2}{a^2x^2}\omega' = \frac{X^2}{x^2}\omega_1. \tag{7}$$

Let $\sigma_x$ = mass per unit area at radius $ax$;
$\bar{\sigma} = m/\pi a^2$.

Then

$$\sigma_x = \frac{dm'}{2\pi a^2 x\,dx}, \tag{8}$$

$$\frac{\sigma_x}{\bar{\sigma}} = \frac{dm'}{2mx\,dx}. \tag{9}$$

On the first hypothesis

$$m' = mX^2, \tag{10}$$

$$x = X^2, \tag{11}$$

$$\omega_x/\omega_1 = 1/x, \tag{12}$$

$$V_x/V_1 = 1, \tag{13}$$

$$\sigma_x/\sigma = 1/2x. \tag{14}$$

On the second hypothesis it can be shown that

$$m' = m\{1-(1-X^2)^{\frac{3}{2}}\}, \tag{15}$$

$$x = \frac{X^4}{1-(1-X^2)^{\frac{3}{2}}}, \tag{16}$$

$$\frac{\omega_x}{\omega_1} = \frac{\{1-(1-X^2)^{\frac{3}{2}}\}^2}{X}, \tag{17}$$

$$\frac{V_x}{V_1} = \frac{\{1-(1-X^2)^{\frac{3}{2}}\}}{X^2}, \tag{18}$$

$$\frac{\sigma_x}{\sigma} = \frac{3(1-X^2)^{\frac{1}{2}}\{1-(1-X^2)^{\frac{3}{2}}\}}{2X^6\{4-(1-X^2)^{\frac{1}{2}}(4-X^2)\}}. \tag{19}$$

The results of these calculations are tabulated for the first hypothesis in Table I and for the second hypothesis in Table II.

It will be seen that, in either case, the density in the central region of the condensation is greater than at the equator and that the angular velocity is higher.

TABLE I

| $x$ | $\omega_x/\omega_1$ | $V_x/V_1$ | $m'/m$ | $\sigma_x/\bar{\sigma}$ |
|---|---|---|---|---|
| 0·9 | 1·1 | 1 | 0·9 | 0·55 |
| 0·8 | 1·25 | 1 | 0·8 | 0·62 |
| 0·7 | 1·43 | 1 | 0·7 | 0·71 |
| 0·6 | 1·67 | 1 | 0·6 | 0·83 |
| 0·5 | 2·00 | 1 | 0·5 | 1·00 |
| 0·4 | 2·50 | 1 | 0·4 | 1·25 |
| 0·3 | 3·33 | 1 | 0·3 | 1·67 |
| 0·2 | 5·00 | 1 | 0·2 | 2·5 |
| 0·1 | 10·00 | 1 | 0·1 | 5·0 |
| 0·01 | 100·00 | 1 | 0·01 | 50·0 |

TABLE II

| $x$ | $\omega_x/\omega_1$ | $V_x/V_1$ | $m'/m$ | $\sigma_x/\bar{\sigma}$ |
|---|---|---|---|---|
| 0·9 | 1·166 | 1·047 | 0·986 | 0·13 |
| 0·8 | 1·365 | 1·094 | 0·957 | 0·23 |
| 0·7 | 1·627 | 1·139 | 0·908 | 0·39 |
| 0·6 | 2·03 | 1·188 | 0·846 | 0·62 |
| 0·5 | 2·48 | 1·239 | 0·762 | 0·92 |
| 0·4 | 3·23 | 1·292 | 0·668 | 1·48 |
| 0·3 | 4·40 | 1·344 | 0·532 | 2·33 |
| 0·2 | 7·1 | 1·395 | 0·383 | 4·2 |
| 0·1 | 14·7 | 1·450 | 0·207 | 10·0 |
| 0·01 | 150·0 | 1·50 | 0·022 | 110·0 |

*Condensation*: *Second phase.*—When contraction of the equatorial radius is checked by rotation, contraction of the polar radius is still possible, and may be expected to continue. The length of the polar radius is related to the pressure of the gas, and a contraction of the polar radius is associated with a fall in temperature.

The material in a rotating cloud, such as that postulated in the previous section, would be too attenuated to admit of the release of nuclear energy, and the release of gravitational energy is determined by the rate of contraction, so that the tempo of the whole process is controlled by the rate at which the heat generated by the contraction escapes into space.

The characteristic features of the second phase are therefore a decreasing polar radius and a falling temperature. Since the process is controlled by the escape of heat, it would no doubt involve a certain lapse of time. In due course the cloud assumes the shape of a disk which becomes slowly thinner and thinner.

Eventually the change of shape leads to instability and we arrive at what may be called the third phase.

*Gravitational instability in a rotating disk.*—The continued existence or otherwise of a local condensation is found to depend on the ratio between the density of the local condensation and the mean density of the system as a whole.

Let $r$ = distance of the condensation from the centre of the disk;
$\bar{\rho}$ = mean density of the system $= 3M/4\pi r^3$;
$a$ = radius of the condensation;
$m$ = mass of the condensation;
$\rho'$ = mean density of the condensation $= 3m/4\pi a^3$.

The problem may be approached in four different ways:

(i) Dealing with a possible condensation in a rotating fluid, Dr Harold Jeffreys gives the formula *

$$\rho' = 0{\cdot}022\bar{\rho}. \tag{20}$$

(ii) Clerk Maxwell has shown † that a ring consisting of $\mu$ satellites of total mass $R$ revolving about a primary of mass $S$ is unstable if

$$0{\cdot}4352\mu^2 R > S. \tag{21}$$

Putting $x = 2\pi r/\mu$ = the distance between the satellites, and assuming that the volume occupied by each satellite is $x^3$, we get

$$\rho' = \frac{R}{\mu x} = \frac{\mu^2 R}{8\pi^3 r^3} \quad \text{and} \quad S = \tfrac{4}{3}\pi\bar{\rho}r^3,$$

so that

$$\rho' > 0{\cdot}038\bar{\rho}. \tag{22}$$

(iii) In the case of a satellite of mass $m$ revolving about a primary of mass $M$ in a circular orbit of radius $r$, it can be shown that a particle may revolve in a closed orbit of radius $a$ about the satellite provided that (approximately)

$$a < r(m/3M)^{\frac{1}{3}}, \tag{23}$$

whence

$$\rho' > 3\bar{\rho}. \tag{24}$$

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* *The Earth*, p. 15, 1929.
† *Scientific Papers*, **1**, 286–387, 1890.

(iv) Roche has shown that a solid satellite of density $\rho_2$, revolving at a distance $r$ about a primary of radius $a$ and density $\rho_1$, is only possible if

$$r > 2{\cdot}44a(\rho_1/\rho_2)^{\frac{1}{3}}. \tag{25}$$

Putting $\rho' = \rho_2$ and $\bar{\rho}r^3 = a^3\rho_1$, this becomes

$$\rho' > 14{\cdot}5\bar{\rho}. \tag{26}$$

It will be observed that the four formulae are dimensionally identical, but that the numerical coefficients are different. As regards (20) and (22) ,the method of comparison is only approximate, since the effect of one ring on another is neglected. A more precise method of comparison would probably show that the two formulae are identical.

There remain then three distinct formulae, that is to say:

(i) Equation (20) which gives the density below which a rotating cloud is known to be stable.

(ii) Equation (24) which gives the minimum density at which it is possible for particles to move in closed orbits about a local centre. When the local density exceeds this figure, local condensations can be regarded as permanent structures.

(iii) Equation (26) introduces the further condition, which is necessary in the case of a solid body, that the angular velocity of the body must be uniform.

The interpretation of the two latter conditions is fairly clear, but the meaning of the first condition is not so obvious. It may represent a change from smooth to turbulent flow.

If these formulae are applied to the case of a disk, such as might be formed in the first and second stages, it is at once apparent that the local density exceeds the minimum values defined by the formulae, and rotation imposes no restriction therefore on the formation of condensations. Applied to the final stage, in which the polar contraction may be less pronounced, they suggest that a certain amount of polar contraction must take place before the birth of a condensation is possible.

It has been suggested that a satellite cannot be formed inside Roche's limit, but this would appear to be somewhat misleading. If we are dealing with a body of lunar mass, it can only exist in the solid state, and a solid body is only possible if the angular velocity is uniform. On the other hand, if we are dealing with a body of stellar mass, the material would be fluid and there is no reason for postulating uniform angular velocity, so that the critical distance within which a satellite can be formed is considerably less. Actually a satellite having the same density as the primary can be formed at an orbital distance of 1·5 times the radius of the primary.

*Condensation*: *Third phase.*—In the original galactic cloud, which has been taken as the starting-point for our discussion, the loss of heat would be so rapid that there would hardly be time to form condensations before the thickness of the disk approached its final value. In the case of a cloud of stellar magnitude, such as we are now considering, however, the rate of cooling would be of an entirely different order, and contraction of the polar axis might require a period of millions of years.

A strict analysis of the development of instability under such conditions would appear to involve difficulties considerably greater than those which are involved in the existing analysis of rotational instability, and for the moment the mathematical problem must be dismissed as insoluble. There are, however, certain points which appear to be deserving of notice.

First, owing to the much slower rate of contraction of the polar axis, conditions would favour the formation of a single satellite rather than a break-up into a large number of small condensations: the latter type of evolutionary process is characteristic of the thin disk rather than the spheroid.

Second, owing to the much greater time available, the effects of viscosity would be significant, and it would appear to be necessary to invoke the existence of viscosity in order to explain the change of shape from the spheroid to a binary system. It may also be noted that small condensations in a viscous fluid tend to coalesce, so that, even if small condensations were formed in the first instance, it is unlikely that they would be permanent.

The inference would appear to be that a rotating cloud whose mass exceeds a certain critical value tends to break up into a large number of small condensations, whereas a cloud of smaller mass tends to form a binary system in which the angular momentum is chiefly associated with the orbital velocity.

*Successive condensations.*—The hypothesis that the stars have been formed by a process of successive condensations appears to meet the objections which apply to the simpler hypothesis that the stars have been formed by a process of direct condensation. The assumption that the first stage is the formation of sub-galaxies meets the difficulty that direct condensation requires a very low temperature, and the assumption that the final stage leads to the formation of binary systems provides for the disposal of the surplus angular momentum.

Nevertheless, the hypothesis of successive condensations is also open to serious objections. The intermediate structures that are postulated in the theory would possess a considerable degree of permanence, and we should expect to find that the majority of the stars were members of star clusters, and that close binary systems were much more frequent than is actually the case.

Since the results of the hypothesis are not in agreement with observation, it must be inferred that the suggested theory is not the chief process that has been responsible for the actual evolution of the stars. In so far as this process has played a part in stellar evolution, it can only have been a minor part, and the main stream of stellar evolution must have followed some other course.

*Cherbury,*
*Booterstown,*
*Co. Dublin:*
1946 *October* 8.

# SOME ASPECTS OF STELLAR EVOLUTION—III

## The Formation of Filaments.

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(Received 1946 October 10)

*Summary*

There are reasons for believing that a rotating disk of gas, when it becomes gravitationally unstable, might break up into filaments or rings rather than spheres or spheroids. It is shown that filaments formed in this way would break up into fragments which might condense to form the actual stars. This hypothesis avoids the main difficulties that were encountered in our discussion of previous theories. The temperature at which the filaments might be formed is not critical and might be 1000° abs. or thereabouts ; the angular momentum of a fragment of the filament is less than the angular momentum of the Sun ; and it is reasonable to suppose that the break up of the filament would give rise to just that mixture of single stars and binary and multiple systems which is actually revealed to us by observation.

It may further be supposed that an embryo star would capture a certain amount of scattered material, and that this material would have a much smaller mass and a much larger endowment of angular momentum than the main body of the star. The bulk of this material might condense to form the planets, and the balance might be absorbed by the star. A theory of this type explains why the rotation of the Sun and the orbital revolution of the planets are in the same sense. It also explains, in a simple and straightforward manner, the peculiar distribution of angular momentum in the solar system.

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*Introduction.*—In conformity with the theories already discussed, it is postulated that the first stage in the evolution of a rotating cloud of gas is the formation of a thin disk. It is further postulated that, as soon as contraction is arrested by gas pressure, the thin disk becomes gravitationally unstable and some further development must be expected. It is at this stage that the various theories diverge.

When a fall in temperature causes water vapour in a closed chamber to condense, the condensations which occur are in the form of small droplets, and it was not unreasonable to assume that condensations due to gravitational instability in a cloud of gas extending in all directions would follow a similar pattern. It was therefore desirable that a trial should be made to see if a similar theory might be applicable to the case of a rotating disk.

The hypothesis that stellar evolution was a process involving the formation of spherical condensations in a rotating disk of gas was examined in Papers I and II, and it was found that attempts to develop this type of hypothesis soon became involved in apparently insurmountable difficulties. In the present paper the problem is approached from a somewhat different angle.

To begin with it may be remarked that we are called upon to discuss the question of instability in a thin sheet of material, and that this constitutes a problem in two dimensions rather than three.

If we observe the waves which are formed on the surface of a sheet of water, it is noticeable that, in general, the waves take the form, not of humps, but of long continuous lines. If a disturbance of this character is a prelude to gravitational instability, then the condensations formed may be expected to be in the shape of filaments or rings rather than in the shape of spheres or spheroids.

It will also be shown that, in a rotating disk, condensation can take place more readily and more quickly in some directions than in others, and the existence of this factor must also be taken into account.

We are led by considerations of this character to the hypothesis that a rotating disk of gas, when it becomes unstable, would probably break up into a number of long parallel filaments. The next step is to show that the filaments would contract upon themselves, and finally that they would become unstable and would break into fragments which might form the actual stars. The purpose of the present paper is to examine some of the implications of this hypothesis.

*Angular Momentum in the Original Disk.*—It was shown in Paper I* that the angular momentum of a point $(x, y)$ about a centre of condensation $C$ is of the form

$$x^2V/r + y^2dV/dr. \tag{1}$$

Under certain conditions $dV/dr$ may be negative, and we may then write

$$\omega = V/r \quad \text{and} \quad \omega' = -dV/dr,$$

so that the angular momentum becomes

$$\omega x^2 - \omega' y^2.$$

It is clear that the angular momentum is zero along the lines

$$\omega^{\frac{1}{2}}x - \omega'^{\frac{1}{2}}y = 0. \tag{2}$$

and

$$\omega^{\frac{1}{2}}x + \omega'^{\frac{1}{2}}y = 0. \tag{3}$$

In the former case the flow of gas is towards $C$, and in the latter case away from $C$. Evidently condensation will take place most readily in the direction defined by equation (2), and particularly so if the time involved is small compared to the period of rotation of the disk. The inference is that gravitational instability will lead to the formation of filaments, and that the axis of these filaments will be at right angles to the direction in which the gas tends to condense.

There are also three other cases to be considered:

When $dV/dr$ is zero the angular momentum in a radial direction is zero, and there will be a tendency to form rings.

When $dV/dr$ is positive and less than $V/r$ the angular momentum is not zero in any direction, but is less in a radial direction than in any other. There will still be a tendency to form rings, but it will be less pronounced.

Finally, if $dV/dr$ is positive and greater than $V/r$, the angular momentum is less in a tangential direction than in a radial direction and there will be a tendency to form radial filaments or bars.

It will thus be seen that there is a good case for the suggestion that the condensations formed in a rotating disk would be in the shape of filaments or rings rather than in the shape of spheres or spheroids.

*Gravitational Instability in the Original Disk.*—Let it be postulated that the temperature in the original disk becomes stabilized at about 1000° abs., and that, being unstable, the disk breaks up into filaments. The arguments which were

* *M.N.*, **106**, 470, 1946.

developed when we were discussing spherical condensations are also applicable in the present case, and it may be assumed that the diameter of a filament is of the same order as the thickness of the disk.

Let $t$ = thickness of the disk;
$\sigma$ = mass per unit area;
$q$ = mass of the filament per unit length;

so that $$q = \sigma t.$$

In Paper I it was shown that *

$$t = 2\mathfrak{R}T/\mu G\sigma;$$

therefore

$$q = 2\mathfrak{R}T/\mu G. \tag{4}$$

Numerically, for $T = 1000°$, $q = 2{\cdot}5 \times 10^{18}$.

It will be noted that this result depends on $T$ only and on the constants $\mathfrak{R}$, $G$ and $\mu$; it does not depend on $\sigma$. It is also interesting to remark that $T$ is apparently determined by the relative proportions of gas and dust, and may be expected to have the same value in different galaxies. It may therefore be expected that the stars in different galaxies would be of the same average mass, and this appears in fact to be the case.

*Lane's Law.*—In the case of a spherical cloud of gas in dynamical equilibrium, density, pressure and temperature can be assumed to be distributed in accordance with a fixed pattern and it is found that certain rules apply during the process of contraction, that is to say

The new radius $r' = \alpha r$;
the new density $\rho' = \alpha^{-3}\rho$;
the new pressure $p' = \alpha^{-4}p$;
the new temperature $T' = \alpha^{-1}T$.

Similar rules are also found to apply for cylindrical and linear contraction.

In the case of a cylinder or filament

The new radius $r' = \alpha r$;
the new density $\rho' = \alpha^{-2}\rho$;
the new pressure $p' = \alpha^{-2}p$;

the temperature $T$ is constant, a result which will be made use of in due course.

In the case of linear contraction as in the case of a disk

The new thickness $t' = \alpha t$;
the new density $\rho' = \alpha^{-1}\rho$;
the new temperature $T' = \alpha T$.

The pressure $p$ is constant, a result which is in conformity with the formulae already given in Paper I.*

*The filament: Dynamical equilibrium.*—The gravitational attraction $g$ at a distance $a$ due to a long filament of mass $q$ per unit length is given by the equation

$$g = 2Gq/a. \tag{5}$$

In the present argument it is assumed that any heat generated in the gas is free to escape, so that $p = k\rho$.

The precise distribution of density and pressure inside a filament of gas is a complex problem which cannot be discussed here. It is sufficient to assume that the gravitational attraction inside a filament of radius $a$ and mass $q$ per unit

* *Loc. cit.*

length is approximately constant and has the value given by the above equation. The density is then given by the formula

$$\rho = \rho_0 e^{-br},$$

and the equation $dp/dr = -g\rho$ gives

$$kab = 2Gq, \tag{6}$$

where $k = \mathfrak{R}T/\mu$.

Now $$q = \int_0^\infty 2\pi p r dr = 2\pi\rho_0/b^2.$$

Putting $\bar{\rho} = \frac{1}{2}\rho_0$ and $q = \pi a^2\bar{\rho}$ we get $ab = 2$.

Therefore

$$q = \mathfrak{R}T/\mu G. \tag{7}$$

It will thus be seen that there is a certain temperature for a filament of any given mass at which there is a balance between the effects of gravity and gas pressure, and that this temperature is independent of the diameter of the filament. If the actual mass of the filament is less than the value defined by this equation, the filament will expand; if it is more, it will contract.

*Contraction of the Filament.*—In discussing the question of stability in the original disk we obtained the formula

$$q = 2\mathfrak{R}T/\mu G,$$

which is in excess of the critical value defined by equation (7). It follows that, once formed, the filament will contract, the rate of contraction being determined by the escape of heat.

The process of contraction involves an increase in density and pressure, and the increased density may be expected to increase the transfer of heat from the gas to the dust particles, thus facilitating the escape of heat and accelerating the rate of contraction. In the early stages of its history it is to be expected that the contraction of the filament will proceed at an ever-increasing rate.

Eventually the increase of density will render the gas opaque, the flow of heat will be checked, the temperature will rise and contraction will be retarded. When this stage is reached, the filament will become longitudinally unstable and will break up into a number of separate parts.

*Angular Momentum in the Filament.*—There is no reason to suppose that there is any tendency for a filament of gas, formed in the manner described, to rotate about its own axis. Angular momentum is entirely in the plane of the disk, and it is of two kinds.

First, the filament as a whole tends to rotate about an axis parallel to the axis of rotation of the disk and the angular velocity is of the order $\omega$.

Now the important point is this. In the case of a filament the process of condensation involves little or no reduction in the length of the filament, and the angular velocity remains substantially constant. Consequently, when the filament eventually breaks up, the angular velocity of rotation is still of the order $\omega$. This result must be contrasted with the case of a spherical condensation in which the process of contraction involves an increase in the angular velocity.

After the filament breaks up the bulk of the angular momentum is in the form of relative motion of the various condensations, and only a small residue is associated with the rotation of the condensations themselves.

Again, angular momentum in the filament may arise from differences in the velocities of the various parts of the filament in a direction parallel to its axis. This relative motion can be regarded as a form of rotation, and in some cases it may be in the opposite sense to the rotation of the filament as a whole.

Whatever the direction, relative motion of different parts of the filament will be opposed by viscosity, and more especially so when the condition of the gas changes from conductive to convective equilibrium. The effect of the slowing down of the longitudinal currents in the filament is to cause a transfer of angular momentum from the system of longitudinal motion to the system of rotation. It may reasonably be assumed that the angular momentum associated with these longitudinal currents is of less importance than that due to the general rotation.

Anticipating the results of the next section and putting $a$, the radius of a condensation when the filament breaks up, equal to $2 \times 10^{14}$ and $\omega$ equal to $10^{-15}$, the angular momentum per unit mass is of the order $4 \times 10^{13}$. This is the equatorial angular momentum, and should be compared with $3 \times 10^{15}$ which is the approximate angular momentum of the Sun at the present time. It will be seen that the angular momentum of the filament accounts for only a small fraction of the total. The possible origin of the remaining angular momentum of the Sun will be discussed in due course.

Meanwhile, it may be noted that the filament theory of stellar evolution disposes of the difficulty of explaining the low value of the Sun's angular momentum which arose in the case of the two previous theories.

*Break up of the filament.*—It has already been pointed out that, once the contraction of the filament has started, the effect of gravity will exceed, for a considerable time, the effect of gas pressure. Eventually, increasing density will delay the escape of heat, the rate of contraction will diminish and the question of longitudinal stability becomes significant.

It is easily shown that a filament of gas which is radially in dynamical equilibrium is longitudinally unstable, so that local condensations will make their appearance and the filament will break up into fragments. These fragments subsequently condense to form the stars.

A point of some interest may be noted in passing. In the early stages of its history, that is to say while it is still transparent, one is justified in postulating conductive equilibrium, but when the filament becomes opaque there must come a change from conductive to convective equilibrium. This change is probably of considerable significance.

If our theory were completely adequate, it should be possible to calculate the dimensions of the filament when the break up actually occurs, but unfortunately our knowledge of the properties of nebular material is not sufficient to justify a precise calculation of this character. We must be satisfied if the theory carries with it no implications which are obviously inconsistent with known facts.

Let $a$ be the radius of the filament when it becomes unstable and breaks up to form the stars. It was shown in an earlier section that the mass of the filament was probably of the order $2{\cdot}5 \times 10^{18}$ gm. per cm. When the break up occurred, it may be assumed that the distance between the condensations was of the same order as the diameter of the filament, so that the mass of a condensation

would be of the order $5 \times 10^{18}\, a$ gm. Let us postulate that the average mass of a star is $10^{33}$ gm. Therefore

$$a = 2 \times 10^{14} \text{ cm.} \tag{8}$$

The opacity of the filament is closely related to its mass per cm.$^2$, and the above value of $a$ gives about $10^4$ gm. per cm.$^2$, equivalent to about 10 atmospheres. This appears to be a reasonable figure.

*The Planets.*—It is not to be supposed that the filaments postulated in our present theory absorbed the whole of the available material; there must have been a certain residue of scattered material which remained unabsorbed when the filaments broke up to form the stars. Some of this material would lie within the sphere of influence of the individual stars and would, in each case, become part of the stellar system.

In accordance with the arguments of Paper I *, stray material in the neighbourhood of a star is moving relatively to the star itself and its angular momentum is of the order $A^2\omega$, where $A$ is the original radius of the cloud of material and $\omega$ is the angular velocity of the star about the centre of the galactic disk ($= 10^{-15}$ for our Galaxy). Owing to its high angular momentum, only a small proportion of this material would be absorbed by the central star and the balance would assume the form of a rotating disk and would be available for the formation of the planets.

A theory of the evolution of the solar system is called upon to answer two questions, among others; first, why the rotation of the Sun is in the same sense as the orbital revolution of the planets, and second, why the angular momentum of the Sun is such a small proportion of the whole. The difficulty is to find a theory which will give satisfactory answers to both these questions at the same time.

The filament theory appears to offer a simple and reasonable solution to the problem. It suggests that the material which now forms the solar system was condensed in two different ways. The bulk of the material, all of which went to form the Sun, was derived from a filament and possessed only a small endowment of angular momentum. The balance of the material, of much smaller mass and much higher angular momentum, was condensed at a somewhat later time from a spherical volume of scattered material which had remained unabsorbed in the formation of the filaments. It was this latter material which went to form the planets and provided the angular momentum of the Sun.

The following calculation is also of some interest. The mass of the planets is about $3 \times 10^{30}$ and their combined angular momentum about $3 \times 10^{50}$. Assuming that the material was derived from a spherical volume of gas whose equatorial angular velocity was of the order $10^{-15}$, we put

$A$ = radius of the sphere,

$\rho$ = density of the gas.

and it is easily shown that

$$A = 5 \times 10^{17} \quad \text{and} \quad \rho = 10^{-24}. \tag{9}$$

This value of $\rho$ is quite consistent with the value of $\rho$ ($10^{-21}$) which was taken as the original density of the gas in computing the size of the filament. Since the numerical data on which these estimates are based are quite unconnected, this agreement appears to be of considerable significance.

* *Loc. cit.*

*The theory reviewed.*—We are now in a position to review the filament theory of stellar evolution.

It will be remembered that the theory that the stars were formed by a process of direct condensation encountered the difficulty that a reasonable estimate of the temperature of the gas is inconsistent with the known masses of the actual stars. The present theory surmounts this difficulty by postulating the formation of intermediate structures in the shape of filaments. It is assumed that the filaments were formed in hydrogen gas at a temperature of about 1000° abs., but this temperature is by no means critical and the theory would be equally effective if it should be found that the correct value of the temperature should be 100° or 10,000°.

Both the earlier theories found difficulty in explaining the low speed of rotation of the Sun. In the present theory this difficulty completely disappears. It is an essential characteristic of the filament theory of stellar evolution that the angular momentum of the individual stars is quite low, indeed definitely less than the angular momentum of the Sun.

It is not to be assumed that the break up of a filament was an orderly process in which condensations were regularly spaced at a uniform distance apart. On the contrary, it may be assumed that a number of incipient irregularities occurred of varying magnitude and importance, and that these disturbances all influenced the final result to a greater or less extent.

It may reasonably be argued, therefore, that a break up of the type suggested would yield precisely the kind of system that is actually observed: a proportion of single stars, and a proportion of binary and multiple systems embracing a wide variety of types extending from close binaries to systems of very wide extent.

The theory that the stars were formed as a result of successive condensations of spherical shape was open to the objection that such intermediate structures would be permanent and would still be in existence if they ever existed at all. A filament, on the other hand, is not a permanent structure and would not be expected to survive for any great length of time. In so far as any trace would remain, it would be in the form of binary and multiple systems exhibiting precisely the characteristic of variety which is actually observed.

It will thus be seen that the filament theory covers, in an adequate and satisfactory manner, the main points with which a theory of stellar evolution may be expected to deal. But it goes further. It provides a satisfactory starting-point for a theory of planetary evolution and explains, without any special or artificial assumptions, how the solar system may have acquired its rather remarkable distribution of angular momentum.

*Cherbury,*
*Booterstown,*
*Co. Dublin:*
1946 *October* 8.

# SOLAR STREAMS OF CORPUSCLES AND THEIR RELATION TO GEOMAGNETIC STORMS

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(Communicated by Professor S. Chapman, F.R.S.)

(Received 1946 September 20)

*Summary*

With a view to study the relation between geomagnetic storms and the solar chromospheric eruptions, the equation of the stream curves of such particles have been obtained as a function of the velocity of emission and the coordinates of the point of emission, generating the two-dimensional motion considered by Chapman. The results obtained show that a very narrow beam of corpuscles emitted from a point on the Sun's surface can produce a magnetic storm of given duration provided the velocity of the emitted particles has a continuous distribution, the width of the velocity spectrum determining the duration of the storm. The velocity of emission of the corpuscles which are possibly responsible for the commencement of some " very great " geomagnetic storms that have occurred in recent years have been calculated. The results obtained show that though the magnitude of the velocity is of the order of $10^8$ cm./sec. as deduced by Milne, the direction of emission should in some cases deviate much from the radial direction, if such storms are to be associated with the solar chromospheric eruptions occurring in the neighbourhood of active sunspots.

Several aspects of geomagnetic disturbance reveal the existence of a close relationship with the Sun, and it seems certain that their main cause is the Sun. The occurrence as well as the commencement of geomagnetic disturbances during the night hours suggest that they are caused by some solar agent that can reach also the dark side of the Earth and hence cannot be the occasional short flares of ultraviolet radiation emitted from the Sun. It is now considered very probable that these disturbances are caused by corpuscular streams emitted from the Sun. The occurrence of a storm in that case will, however, depend on the velocity of the emitted corpuscles and also on the relative positions of the Sun and the Earth. Instances are, however, not rare when a magnetically disturbed day is followed after one synodic rotation period of the Sun, by a magnetically calm day, which shows either that there were no solar emissions at a time corresponding to the second epoch, or that the velocity of the emission of the corpuscles had altered.

Although continuous observations on the conditions of the Sun are not yet possible there is some visible evidence to show that corpuscular streams are emitted from the Sun with considerable velocities, though very little is as yet known about the process due to which the Sun ejects matter from its surface. Pettit * concluded from a study of the solar prominences that the solar corpuscles before emission are subjected to repeated sudden impulses which cause an accelerated motion after emission. Milne † showed from theoretical considerations that if the selective radiation pressure is mainly responsible for the support of the higher levels of the solar atmosphere, then it is possible that solar corpuscles will be accelerated away

* *Ap. J.*, **84**, 319, 1936.
† *M.N.*, **86**, 459, 578, 1926.

from the Sun, and they can acquire in that way a limiting velocity of the order of $1{\cdot}6 \times 10^8$ cm./sec. While this type of emission appears likely to occur, no definite observational evidence has been obtained which lends strong support to this view. Pike * has shown that together with Milne's process of emission there is the possibility of horizontal motion of the particles near the surface of the Sun, caused by local temperature inequalities or by an abnormal horizontal component of atomic motion. On this basis he has shown that sunspots must act like centres of attraction to prominence matter in the surrounding region, causing matter to be drawn towards them as is often observed. It has also been shown that those solar corpuscles which acquire large horizontal velocities are accelerated both in the horizontal as well as the vertical direction, and are liable to leave the Sun altogether.

Geomagnetic storms are very often associated with sunspot activity. Greaves and Newton † and Newton ‡ have shown from statistical considerations that the magnetic storms which can be classified as "very great" are almost always associated in some way with the appearance of an active sunspot near the Sun's central meridian. It has also been shown that, in a great majority of cases, the occurrence of large storms is associated with the presence of large spots in a sector between 26° E. and 53° W. of the central meridian at the time of commencement of the storm. The average position, however, is a meridian which at that instant has a longitude of about 14° W. of the central meridian. It is thus probable that all the geomagnetic storms classed as "very great" are produced by solar corpuscles ejected from the Sun as a result of chromospheric eruptions occurring in the region surrounding an active sunspot. Although the mechanism by which these eruptions take place is not yet known definitely, on the assumption that they are coming out with a definite velocity, their trajectory can be easily determined and the conditions under which they can produce geomagnetic storms can be studied. Chapman§ has determined the nature of such streams of solar corpuscles on a very simplified model, and has discussed the conditions under which these corpuscles can produce magnetic storms. The values derived by him are valid only when the solar corpuscles are ejected from the Sun's equator and the tangential velocity of the emitted particles is equal to that of the Sun's surface at the point of emission. He has, however, also discussed the possibility of a storm being produced when the sunspots lie in higher heliographic latitudes, though following Milne he has assumed that the average direction of emission of corpuscles even in higher latitudes is radial. The trajectory discussed by him is a curve in two dimensions although in reality the trajectory is a curve in three dimensions. In the present paper the nature of the trajectory of solar particles emitted from different heliographic latitudes and with different velocities is determined and applied to study the conditions under which such a stream can produce a geomagnetic storm.

Let $a$ denote the Sun's radius, $a$, $\alpha$, $\beta$, the spherical polar coordinates, referred to the centre and axis of the Sun, of a point $P$ on the surface of the Sun at which the chromospheric eruption has occurred. Let $\gamma$, $\theta$, $\phi$, be the coordinates, at any time $t$, of the particle which was emitted from $(a, \alpha, \beta)$ at time $t_0$. The law of force under which these ejected particles will move is not known definitely.

* *M.N.*, **88,** 3, 635, 1928.
† *M.N.*, **88,** 556, 1928.
‡ *Observatory*, **62,** 318, 1939.
§ *M.N.*, **89,** 456, 1929.

Milne's hypothesis leads to a central force varying inversely as the square of the distance and Chapman has shown that in that case even at a distance $\gamma = 10a$, the velocity of the particles is 94 per cent. of the velocity they will acquire in travelling an infinite distance. Consequently over much the greater part of their journey from the Sun to the distance of the Earth these particles can be considered as moving with a uniform velocity. It is therefore possible, without introducing any appreciable error in the calculations of the present paper, to consider that these particles after emission are subjected to no external force.

Let the solar axis be taken as the axis of $z$, and the equatorial plane as the plane of $x$, $y$. For convenience let us assume that the point $P$ at time $t = 0$ lies in the $zx$-plane. This then can be suitably defined with reference to any fixed meridian plane. Let $u_1$, $u_2$ be the components of the velocity of emission of the particles emitted at time $t_0$ along and perpendicular to the axis of rotation in the meridian plane passing through $P$ at time $t_0$, and $u_3$ that in a direction perpendicular to the meridian plane. Considering the motion of a particle in three dimensions and under no external force we have

$$\gamma \cos\theta = u_1(t - t_0) + a\cos\alpha, \tag{1}$$

$$\gamma \sin\theta \cos(\phi - \Omega t_0) = u_2(t - t_0) + a\sin\alpha, \tag{2}$$

$$\gamma \sin\theta \sin(\phi - \Omega t_0) = u_3(t - t_0), \tag{3}$$

where $\Omega$ denotes the angular velocity of the Sun, which according to Carrington is given by the relation

$$\Omega = \frac{2\pi}{25{\cdot}4} = 0^\circ 35'{\cdot}43 \text{ per day.}$$

If therefore $(x, y, z)$ be the coordinates at time $t$ of the particle emitted at time $t_0$, then

$$x\cos\Omega t_0 + y\sin\Omega t_0 = u_2(t - t_0) + a\sin\alpha, \tag{4}$$

$$y\cos\Omega t_0 - x\sin\Omega t_0 = u_3(t - t_0), \tag{5}$$

$$z = u_1(t - t_0) + a\cos\alpha. \tag{6}$$

If it is assumed that the corpuscles are emitted from the point $P$ with a constant velocity relative to the axis and the meridian plane, while $P$ itself is moving with uniform angular velocity $\Omega$, then it can easily be shown that at any time $t$ all the particles emitted before that instant will lie on the curve given by

$$x^2 + y^2 - \frac{u_2^2 + u_3^2}{u_1^2}\left(z - a\cos\alpha + \frac{u_1 u_2}{u_2^2 + u_3^2}\, a\sin\alpha\right)^2 = \frac{u_3^2}{u_2^2 + u_3^2}\, a^2 \sin^2\alpha$$

and (7)

$$x\cos\Omega\left(t - \frac{z - a\cos\alpha}{u_1}\right) + y\sin\Omega\left(t - \frac{z - a\cos\alpha}{u_1}\right) - u_3\left(\frac{z - a\cos\alpha}{u_1}\right) = 0.$$

The form of the stream at any time $t$ is thus a curve in three dimensions, and depends on three parameters $u_1$, $u_2$, $u_3$. Different values of $u_1$, $u_2$, $u_3$ give different stream curves, and if $u_1$, $u_2$, $u_3$ vary continuously within definite limits all the corpuscles at time $t$ will lie within a conical tube with its vertex at $P$; the nature of its surface will depend on the limits within which the values of $u_1$, $u_2$, $u_3$ vary. The variations in $u_1$, $u_2$, $u_3$ may be due simply to a change in the direction of emission of the corpuscles without any change in absolute magnitude, or due to a change in magnitude without any change of direction, or both. Chapman

considered only the first case, and restricted the motion to two dimensions. He also assumed that the emitting area has a finite extension both in latitude and longitude, depending on the duration of the storm.

It is now easy to calculate the velocity components $u_1$, $u_2$, $u_3$ of the emitted particles which on emission at time $t_0$ will reach the Earth at time $t$. If $R$, $\Theta$, $\Phi$, be the coordinates of the Earth at the instant when the storm commences, and if $t_0$ be the time at which the particles responsible for the first impulse were emitted, then we have from (1), (2) and (3)

$$R\cos\Theta = u_1 T + a\cos\alpha, \tag{8a}$$

$$R\sin\Theta\cos(\Phi - \Omega t_0) = u_2 T + a\sin\alpha, \tag{8b}$$

$$R\sin\Theta\sin(\Phi - \Omega t_0) = u_3 T, \tag{8c}$$

where $T$ is the interval between the time of eruption and the time of commencement of the storm, and is equal to $t - t_0$, where $t$ is the time corresponding to the commencement of the storm; $t_0$ can be taken to represent the instant when the solar eruption was at its maximum. The durations of chromospheric eruptions are usually small, and so the uncertainties in the value of $t_0$, in calculating $T$, will not appreciably affect the estimated values of the velocity components. It is now clear that for given $T$ and $t_0$, the values $u_1$, $u_2$, $u_3$ are practically independent of $\alpha$. In Table I, I have given the values of $u_1$, $u_2$, $u_3$ for some different values of $\Theta$ and $\Phi - \Omega t_0$, taking $T = 24$ hours. For other values of $\Theta$ and $\Phi - \Omega t_0$, the corresponding values may also be obtained from this table by interpolation or otherwise.

TABLE I

*Velocity components in units of* $10^8$ *cm./sec. corresponding to* $T = 24$ *hours*

| | $u_1$ | | $u_2$ | | $u_3$ | |
|---|---|---|---|---|---|---|
| $\Phi - \Omega t_0$ \ $\Theta$ | 90° | 83° | 90° | 83° | 90° | 83° |
| 0 | 0 | 0·212 | 1·74 | 1·73 | 0 | 0 |
| 5° | 0 | ,, | 1·73 | 1·72 | 0·151 | 0·151 |
| 10° | 0 | ,, | 1·72 | 1·70 | 0·302 | 0·300 |
| 20° | 0 | ,, | 1·64 | 1·62 | 0·596 | 0·591 |
| 40° | 0 | ,, | 1·33 | 1·32 | 1·12 | 1·11 |
| 60° | 0 | ,, | 0·871 | 0·864 | 1·51 | 1·50 |

Since the velocity components corresponding to given values of $\Theta$ and $\Phi - \Omega t_0$ are practically inversely proportional to $T$, their values corresponding to different values of $T$ can also be obtained from Table I. If the magnetic storm is supposed to continue as long as the Earth is enveloped in the stream of corpuscles, then the breadth of the stream where they cross the Earth's orbit should vary with the duration of the storm. Chapman has shown how this breadth depends on the extent of the area on the surface of the Sun, from which the emission takes place, and on the thermal motions of the corpuscles of the stream, though the latter makes a

very small contribution. He has assumed that the velocity of emission is always constant in magnitude. In some cases when the duration of the storm is 24 hours or more, according to Chapman, it will be necessary to consider that the emitting area on the surface of the Sun has a large angular breadth, which is unlikely. It can now be seen that such a large breadth of the stream can also be produced by a very narrow beam of corpuscles emitted from a point, provided only that the velocity of the emitted particles vary in magnitude continuously over a range, the limits of which depend on the duration of the storm. If therefore the duration of the storm be taken to be $\tau$, then we can easily find the lower limits of the velocity components from equation (8) by replacing $T$ by $T+\tau$ and $\Theta$, $\Phi$ by $\Theta'$, $\Phi'$, their values corresponding to the end of the storm, assuming that the corpuscles responsible for the beginning and end of the storm are emitted simultaneously. Although it cannot be claimed as yet that there is definite evidence to show that a continuous distribution within finite limits exists for velocities of these solar corpuscles, Pettit has suggested that the velocities increase by jerks during the period of emission.

To obtain a visual picture of the stream, and also to compare the corresponding results given by Chapman, we now deduce from equations (4)–(6) the results for the case when the motion is reduced to that in two dimensions. We assume that $u_1$ is zero, so that the stream of corpuscles will always lie in a plane parallel to the Sun's equator. The stream curve at time $t$ is thus given in terms of a single parameter $t_0$ by the following equations

$$x=(u_2\cos\Omega t_0-u_3\sin\Omega t_0)(t-t_0)+a\sin\alpha\cos\Omega t_0, \qquad (9a)$$

$$y=(u_2\sin\Omega t_0+u_3\cos\Omega t_0)(t-t_0)+a\sin\alpha\sin\Omega t_0, \qquad (9b)$$

$$z=a\cos\alpha. \qquad (9c)$$

The case discussed by Chapman can be deduced from the above result if we take $\alpha=90^\circ$ and $u_3=a\omega_0$, and then the corresponding results will agree approximately * with that given by Chapman. (Equations (6), (8) and (14) of Chapman can be deduced from the above after making the necessary approximations.) For illustration and comparison we consider a particular case when the emitting point is on the Sun's equator, so that $\alpha=90^\circ$. In that case the solar corpuscles will always continue to be in the Sun's equatorial plane, and the equation of the stream curve can easily be obtained from equations (9). In Fig. 1, I have drawn the curves for some particular values of $u_2$, $u_3$ and $t$. AB represents the curve for $t=24$ hours, $u_2=1{\cdot}64\times10^8$ cm./sec., $u_3=0{\cdot}60\times10^8$ cm./sec., whereas CD and C′D′ represent the corresponding curves for $u_2=0{\cdot}82\times10^8$ cm./sec. ; $u_3=0{\cdot}30\times10^8$ cm./sec. and $t$ equal to 24 and 48 hours respectively, assuming that the emission of the corpuscles has continued for 24 hours. The five marked positions on the different stream curves correspond to the positions of the corpuscles emitted at exactly 24, 21, 18, 10 and 2 hours respectively before the commencement of the storm. The actual chromospheric eruption, however, has a short duration ; the space occupied

* Chapman has shown that under a central force $\dot\gamma=f(\gamma)$, and that if the particles are subject to no force after emission $f(\gamma)=v_0$. This is not correct unless $\omega_0=0$. In the case considered by him, it can be shown that $\dot\gamma^2=v_0^2+a^2\omega_0^2\left(1-\frac{a^2}{\gamma^2}\right)$. Since the factor $\left(1-\frac{a^2}{\gamma^2}\right)$, is always less than 1 and approaches unity as $\gamma$ increases and also $v_0\gg a\omega_0$, in the case under consideration $\dot\gamma$ can be taken to be equal to $v_0$ *approximately*.

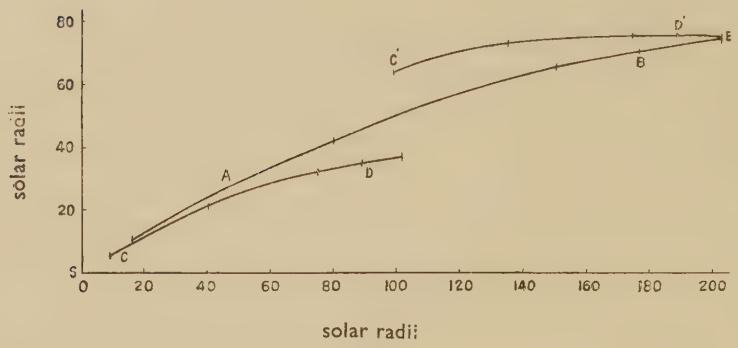



FIG. 1.

by the corpuscles in such cases can, however, be easily obtained from the above figure. The unit of distance has been taken as the Sun's equatorial radius, and S and E represent the positions of Sun and Earth. The present analysis shows that if the point of emission be properly situated relative to the Earth, then AB will correspond to the stream of corpuscles which is responsible for the commencement of the storm when the storm commences just 24 hours after the observed eruption on the surface of the Sun. CD and C'D' will then represent the positions of the particular stream of corpuscles, at the beginning and end of the storm, which is responsible for the end of the storm if the duration of the storm is taken to be 24 hours, and the post perturbation effect is neglected in calculating the duration of the storm. The above values of the velocity components give an idea as to the limits within which the magnitude of the velocity components should vary if a storm is to commence after 24 hours from the instant when the eruption took place, and if the storm continues for 24 hours. For other values of $t$ and $T$ similar results can be easily obtained.

At a large distance from the Sun, for which $\gamma \gg a$, the stream lines can be represented approximately by the equation

$$\gamma = \frac{(u_2^2+u_3^2)^{\frac{1}{2}}}{\Omega}\left\{\Omega t - \theta + \tan^{-1}\frac{u_3}{u}\right\}. \tag{10}$$

Consequently if $\chi$ represents the inclination of the stream to the radius through the point $(\gamma, \theta)$, then for $\gamma \gg a$, we have

$$\tan\chi \approx -\frac{\Omega\gamma}{(u_2^2+u_3^2)^{\frac{1}{2}}}. \tag{11}$$

The values obtained by Chapman can be deduced from the above with the corresponding values of $u_2$ and $u_3$. Since $\chi$ is always negative, it shows that (as already indicated by Chapman) if the stream of corpuscles are undeflected by the Earth's magnetic field it will first strike the Earth's atmosphere on the post-meridian side of the sunlit hemisphere on a meridian which differs from the twilight meridian by an angle depending on the velocity of the corpuscles at emission. In the present paper we have altogether ignored the possible deflection of the corpuscles by the Earth's magnetic field. It is possible that due to the Earth's field the corpuscles will actually impinge on the Earth on a meridian slightly differing from the one deduced from (11). Moreover it is also probable that the storm will commence sometimes before the stream actually strikes the surface of the Earth.

We shall now consider some particular cases of magnetic storms that have occurred in recent years which, according to the records of Alibag Observatory have been classified as "very great", and are also associated with the appearance of active sunspots. Although the time of commencement of the storm can be determined in many cases quite accurately, the instant at which the particles which first impinge on the Earth were emitted cannot be exactly determined from observations. The duration of observed chromospheric eruptions in general is small, varying roughly between 20 to 40 minutes. Consequently the estimate of travel time will not be very much in error if we take the time of emission of those particles responsible for the commencement of the storm as that given by the middle of the interval during which the eruption took place, or the time at which the eruption was at its maximum. Moreover, in the neighbourhood of an active sunspot, eruptions take place at close intervals, and purely solar observation is not sufficient to determine which one is responsible for the magnetic storm recorded. To obtain an estimate of the velocity components of the corpuscles responsible for the commencement of a particular storm according to the results deduced here, we have chosen some particular eruptions which according to their intensity and position are most likely to be associated with the storm. The necessary data pertaining to the conditions on the surface of the Sun have been taken from the *Kodaikanal Observatory Bulletin* * and the *Quarterly Bulletin on Solar Activity of the International Astronomical Union.* The results obtained are given in Table II, where $T$ represents the travel time of those corpuscles which are responsible for the commencement of the storm, $\alpha$ the co-latitude of the point where the

TABLE II

| Date of storm | $T$ in hrs. | $\alpha$ in degs. | Vel. components in $10^8$ cm./sec. | | | | $\psi$ in degs. | $t_c$ in hrs. |
|---|---|---|---|---|---|---|---|---|
| | | | $u_1$ | $u_2$ | $u_3$ | $V$ | | |
| 1940 | | | | | | | | |
| Mar. 24 | 50 | 78 | −0·10 | 0·33 | 0·76 | 0·83 | 68 | −71·5 |
| 24 | 31·5 | 78 | −0·16 | 1·02 | 0·83 | 1·32 | 43 | −39 |
| 24 | 26·5 | 78 | −0·19 | 1·16 | 1·05 | 1·58 | 46·1 | ,, |
| 1941 | | | | | | | | |
| Mar. 1 | 51·3 | 74 | −0·10 | 0·81 | 0·060 | 0·82 | 23·7 | 44·7 |
| 1 | 32 | 74 | −0·17 | 1·29 | −0·14 | 1·31 | 24·4 | ,, |
| July 5 | 46·5 | 75 | −0·063 | 0·88 | 0·16 | 0·90 | 15·7 | 33·8 |
| 5 | 35 | 78 | 0·083 | 1·19 | 0·10 | 1·20 | 9·3 | ,, |
| 5 | 25·3 | 78 | 0·115 | 1·65 | −0·086 | 1·66 | 9·4 | ,, |
| Sept. 18 | 38 | 77 | 0·13 | 1·07 | 0·17 | 1·09 | 10·5 | 25·7 |
| 18 | 25 | 80 | 0·20 | 1·65 | −0·087 | 1·661 | 4·6 | ,, |
| 18 | 19·8 | 83 | 0·26 | 2·06 | −0·33 | 2·10 | 8·6 | ,, |
| 1946 | | | | | | | | |
| Feb. 7 | 76·4 | 65 | −0·062 | 0·478 | 0·255 | 0·542 | 41·3 | 32·7 |
| 7 | 27·8 | 65 | −0·175 | 1·49 | −0·0521 | 1·50 | 31·7 | ,, |
| Mar. 28 | 26·5 | 70 | −0·126 | 1·04 | 0·113 | 1·05 | 27·2 | 28 |

* I am indebted to Dr A. L. Narayan, Director, Solar Physics Observatory, Kodaikanal, for supplying me the relevant solar data used in this paper in connection with the storm of 1946 March.

eruption occurred and $V$ the resultant velocity of emission of the corpuscles; $\psi$ is the inclination of the direction of emission with the solar radius through the point of emission; $t_c$ is the interval of time between the crossing of the central meridian by the associated sunspot and the instant of commencement of the storm, and is considered positive when the storm commences after the sunspot has crossed the central meridian. The values corresponding to the storm that started on 1946 February 7 have also been included. From a consideration of range it cannot be classified as very great, though it was remarkable for causing rapid fluctuations in all the three magnetic elements for a long time, and was possibly associated with the great sunspot which was at a high latitude of 25° N. The time of eruption in this case, however, was taken from the records of distinct solar flares as recorded in the magnetograms, and the reported sudden commencement of radio fade-outs. This will possibly almost synchronize with the intense and large solar eruptions that have taken place. The results given above show that Chapman's simplified theory cannot explain the storm of 1940 March. The large values of $\psi$ in the case of several storms show that at least in those cases the mean direction of emission of the particles is not likely to be radial.

There is, however, evidence to show that magnetic disturbances have occurred at times when no sunspots were visible for days. On the other hand there is evidence to show that active sunspots have crossed the central meridian without producing any disturbance. But these anomalies can be easily explained if we remember that sunspots themselves are not the cause of the storms but are only associated with the cause. This also explains the marked absence of the 27-day recurrence tendency for the "very great" magnetic storms. Sunspots are often associated with magnetic storms, and that can be explained if we remember that they are always associated with bright areas from which corpuscles are emitted in spite of the attraction exerted by the spots themselves. A proper estimate of the velocity of the corpuscles is not possible unless the time of commencement of eruption as well as its location on the surface of the Sun can be obtained definitely. When these become known from the results of solar observations the analysis given in this paper will help to determine the geometry of the stream of corpuscles. Milne's hypothesis leads to a maximum velocity of $1{\cdot}6 \times 10^8$ cm./sec. for the particles, but this cannot explain also the observed lower level of the auroral rays, which require either that the velocity of these corpuscles is very high (for which some undiscovered mechanism is responsible) and is comparable to that of light, at least in the neighbourhood of the Earth, or that the Earth's atmosphere is much more penetrable than is indicated by our present knowledge about the constitution of the upper atmosphere. The electrical state of these corpuscles has also not been taken into consideration, but various considerations show that the stream is neutral but ionized. An electrically charged stream emitted from the Sun producing direct magnetic action and thereby causing magnetic disturbance on the Earth is not possible unless the corpuscles have velocities nearly equal to that of light. In that case, however, the corpuscles will be highly penetrating and tend to reduce very much lower limits of auroral light; also they should cause a large variation in the intensity of cosmic rays during magnetic storms which, however, is not fully supported by experimental evidence. The results obtained in the present paper show that for the occurrence of a storm it is sufficient if the resultant velocity of the emitted corpuscles is of the order $10^8$ cm./sec., but they should in some cases be

emitted at a large angle with the radial direction. The Doppler shift of the different lines in the solar spectrum observed at suitable intervals during the period when a geomagnetic storm is in progress will possibly determine only the average velocity of the group of particles composing the stream and not the actual distribution of the velocity of the corpuscles if there be any.

The results given in Table I show that $u_1$, $u_2$, $u_3$ are practically independent of $\alpha$ and $\Theta$. This agrees with the results obtained by Bartels * who has shown from a statistical consideration that the incidence of magnetic disturbance is independent of $\alpha$ and $\Theta$. Chapman's simple theory, however, requires that in September (when $\Theta > 0$) the magnetic disturbance is mainly due to northern solar emitting areas ($\alpha < 90°$), and in March (when $\Theta < 0$) to southern areas ($\alpha > 90°$), which is not supported by the evidence obtained by Bartels and others.

*Alibag Observatory,*
*Bombay:*
1946 *September* 10.

* *Terr. Magn. Atmos. Elect.*, **37**, 1, 1932.

# VISUAL AND SPECTROGRAPHIC OBSERVATIONS OF A GREAT SOLAR FLARE, 1946 JULY 25

*M. A. Ellison*

(Received 1946 September 26)

*Summary*

Measures of effective line-width of the bright *Hα* emission support previous results in bringing to light the existence of what may be described as a *radiation burst* at the commencement of a great flare. Its duration is of order of 2–3 minutes only, as compared with a total duration for the flare of several hours.

Seven flare spectra were obtained, covering the period before and after the time of maximum intensity at $16^h\ 27^m$. One plate, exposed within one minute of the time of peak intensity, is of special interest, and the following unusual features have been recorded :

(*a*) Reversal of the *Hα* line 15 A. wide, with a central intensity greater than 300 per cent. of the continuous spectrum of the photosphere. The wings of the bright emission show an asymmetry which has been noted at Sherborne in all previous flares.

(*b*) Helium λ 6678 in strong emission. Helium λ 5876 in emission over the flare and in absorption over the penumbra of the sunspot as well as over the surrounding photosphere.

(*c*) Reversals of λλ 6347·1, 6371·4 of ionized silicon.

(*d*) Contours of many iron and nickel lines filled in.

(*e*) An enhancement of the continuous spectrum over a small area which coincides in position with the brightest part of the flare filament as seen by light of *Hα*. This phenomenon appears on only one plate and is simultaneous with the radiation burst. It seems probable that we have here the explanation of the phenomenon observed by Carrington and Hodgson in integrated light on 1859 September 1.

These observations all point to the need for continuous spectrographic records at short intervals, especially during the first few minutes of flare activity.

1. *Introduction.*—During the early months of 1946, my spectrohelioscope, which had been in use since 1939 for the visual recording of flares, was reconstructed in a form designed to permit the rapid photography of flare spectra.

Continuous measures of the *effective line-width* of the bright *Hα* emission, made during the course of observing a number of flares, were published in an earlier paper.* From the shape of these curves, showing line-widths against time, it was inferred † that, shortly after the commencement of a great flare, there occurs a sudden, catastrophic outburst of emission, the effect of which is to fill up the normal contours of hydrogen, superimposing upon them a brilliant inverted contour, whose bright wings may extend 10 A., or more, on either side of the normal line centre. Thereafter, the line-width and the intensity of the bright emission die away, rapidly at first and then more slowly. The duration of this initial *burst of radiation* is of the order of 2 to 3 minutes only, while the total life of a great flare may be as many hours.

* *M.N.*, **103**, 3, 1943.
† *Observatory*, **65**, 116, 1943.

It was also anticipated that the most remarkable features of flare spectra would be found to coincide with the very short period of the radiation burst, as would the commencement of those immediate geophysical effects which depend upon the reception in the ionosphere of ultra-violet light, namely the magnetic "crochet" and the short-wave radio fade-out. Speed is thus essential in hunting for flare spectra, and the spectrograph must always be in readiness when there are large sunspots on the disk. Accurate timing is important; for it will be apparent that some of the most unusual spectral features on 1946 July 25 were confined to a single plate taken within one minute of the peak intensity of the flare.

2. *Apparatus and observing methods.*—With the new instrument, a flare seen to be commencing on the disk is centred in the field of view of the spectrohelioscope and the oscillating slits are brought to rest. The stationary scanning slit now passes light to the grating from a narrow strip in the image of the flare by integrated light. The grating can then be rotated by means of a handle, which operates a gear-wheel attached to the grating table, so that any region of the first, second or third order spectrum, designated by a wave-length scale, may be brought to bear on a 10 in. × 2 in. plate-holder adjacent to the slit. The linear scale of the plate is 4·4 A./mm. in the first order. Exposures, of about 2 seconds' duration, are made with a simple flap shutter, after which the $H\alpha$ line in the first order may be returned to the viewing slit of the spectrohelioscope, so that the progress of the flare may be followed visually with but little interruption. After some practice, this series of operations can be completed within 30 seconds.

The present optical system of the spectrohelioscope incorporates two 4-inch single lenses, as objective and collimator respectively, the plate-holder being usually adjusted to provide accurate focus in the region between $H\alpha$ and the D lines. For other spectral regions refocusing is required and this leads to delay, but it is hoped shortly to replace the single lenses by an achromatic system. Experience gained over a period of three months suggests that the combined spectrohelioscope and spectrograph will prove a convenient instrument for recording the spectra of these unexpected and transient phenomena.

3. *Visual observations.*—After several days of exasperating weather, the sky cleared at about $15^h$ on July 25 and there followed until sunset a period of very fine seeing, such as is usually experienced only in the early morning hours. At $15^h\ 13^m$ a Class 2 flare was seen to be in progress, in the form of a bright filament traversing the great spot group in the direction north-west to south-east, along what may be called the *flare axis* of the spot: this had reverted to normal intensity by about $15^h\ 27^m$.

On returning to the instrument at $16^h\ 15^m$ a new flare was seen to be commencing in the same position as the former, but already more brilliant and extensive. Two bright, sinuous filaments now lay across the group and extended some distance beyond the boundaries of the penumbra. These brightened rapidly, and were judged to be most intense when observed about $16^h\ 30^m$, at which time their enormous extent (250,000 km. in length) and "white-hot", appearance could only be described as the most impressive solar phenomenon (Fig. 1) observed hitherto with the Sherborne spectrohelioscope. The area of the filaments, measured with a graticule, was found to be 1,270 millionths of the visible hemisphere, or about one-third the area of the spot group. In the intervals between photographing the spectrum, sketches were continued and measures for

effective line-width of the *Hα* emission were made with the line-shifter until $18^h\ 10^m$.

The bright filaments A and B (Fig. 1A) retained their identity throughout the whole period, A elongating south-eastwards and B north-westwards along parallel paths with only a narrow division between. At $16^h\ 56^m$ the appearance was that shown in Fig. 1B: the filaments were now less intense and more widely diffused than at $16^h\ 30^m$. By $17^h\ 30^m$ (Fig. 1C) the area of the bright emission had increased to 2,500 millionths and the filaments were spread over a length of 550,000 km., some extensions reaching out almost to the stable dark flocculi which were located

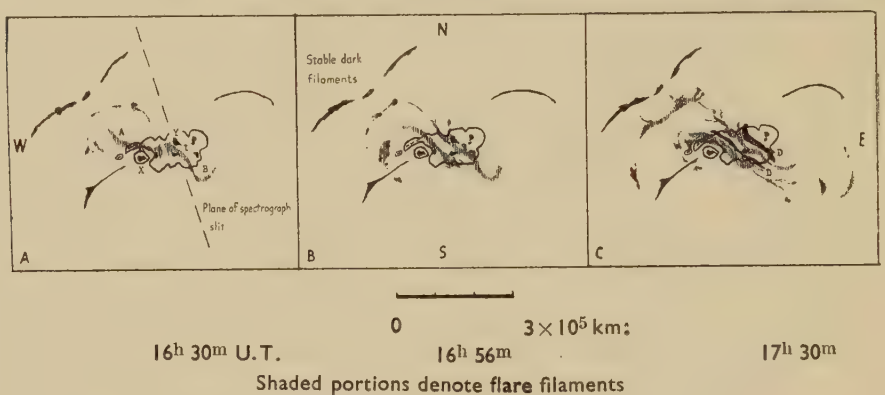



FIG. 1.—*Spectrohelioscope drawings showing the progress of the Great Flare by light of the Hα line.*

*A. Area of bright filaments*=1270 *millionths. Length* (*A-B*)=230,000 *km.*

*Note the sharp central division between the filaments A and B.*

*X, Y, spot umbrae appearing in flare spectra* (*Plate* 13).

*C. Area of bright filaments*=2500 *millionths.*

*A general outward diffusion has taken place and new filaments have formed E and W. D, D, descending dark filaments*; 75 *km./sec. at* $17^h\ 50^m$; 70 *km./sec. at* $18^h\ 05^m$.

250,000 km. north-west of the spot group. From $17^h\ 30^m$ onwards, dark *descending* absorption flocculi (+75 km./sec. at $17^h\ 50^m$) were seen on the northern boundaries of the two bright filaments where these crossed the spot area (see also Section 5). No absorption markings in *ascent* were seen during the early stages of the flare, as is customary, but it may be that preoccupation with the spectrograph at this period caused them to pass unnoticed. The sketch in Fig. 1C coincides closely in time with M. d'Azambuja's excellent spectroheliogram in *Hα* taken at Meudon, $17^h\ 32^m$, but on account of the Doppler displacements, we should not expect these high-velocity dark filaments to appear on the latter.

4. *Effective line-width of Hα emission.*—In Fig. 2A we have a plot of the line-width measures extending from $16^h\ 20^m$ to $18^h\ 08^m$, and relating to the *Hα* emission coming from the filaments in the centre of the spot group. viz. between the principal umbrae X and Y of Fig. 1A. The method of measurement was the same as that described in a former paper *, where it was shown that the variations of *Hα* line-width with time provide a simple quantitative method of following the course of a flare, the line-width being a function of the total emission from the inverted bright contour.

From the shape of this curve, the peak of the radiation burst can be estimated

* *Loc. cit.*

with some confidence to have occurred at $16^h\ 27^m \pm 1^m$. An increase in line-width from 8 A. to 15 A. seems to have taken place in about 3 minutes and the subsequent decline was only a little less rapid. From $16^h\ 40^m$ to $18^h\ 08^m$ the decline was steadier and more nearly proportional to time. The maximum line-width of 15·2 A., which occurred at $16^h\ 27^m$, is a very conservative estimate * derived from the second spectrum plate, but it is possible that a much greater value may have been momentarily attained in the interval between $16^h\ 23^m$ and $16^h\ 29^m$. Observarions of solar radio noise emissions at a wave-length of 4·7 metres, recorded by J. S. Hey and his colleagues at the research station of the Operational Research Group, Ministry of Supply, showed a peak of extremely large amplitude between $16^h\ 24^m$ and $16^h\ 32^m$.†

A brief review of the other geophysical effects associated with this flare has been given by H. W. Newton in *The Observatory*.‡

TABLE I

*Flare Spectra*

| Index letter | Range of wave-length λ – λ | Order of spectrum | Filter over slit | Time U.T. h m |
|---|---|---|---|---|
| A | 6300 – 6750 | R 1 | R.G. 2 | 16 25 |
| B | 6000 – 6800 | R 1 | O.G. 2 | 16 27 |
| C | 5700 – 6750 | R 1 | O.G. 2 | 16 35 |
| D | 6300 – 6800 | R 2 | R.G. 2 | 16 37 |
| E | 6300 – 6800 | R 2 | R.G. 2 | 16 50 |
| F | 4950 – 6100 | R 1 | None | 16 52 |
| G | 5650 – 6700 | R 1 | O.G 2 | 17 15 |

5. *Spectrographic records.*—Table I contains the relevant details of the spectra taken during the course of the flare. All the exposures were made on Kodak III—F emulsion. The times of each are indicated and they will be referred to by the index letters A, B, C, etc.

In every case the spectrograph slit was parallel to the broken line in Fig. 1A. In exposures B, D, E and G the slit was made to bisect the umbra marked Y, immediately south of which was situated one of the most brilliant patches of flare emission. In A, C and F the slit passed across or near to the umbra X.

Spectrum B (Plate 13) is of special interest, having been exposed probably within one minute of the peak intensity of the flare. The lines on this plate, appearing either with reversed contours or merely as absorption lines partly filled in, are listed in Table II.

*Hydrogen.*—The *Hα* emission is brighter than the neighbouring continuous spectrum of the photosphere on all plates taken between $16^h\ 25^m$ and $17^h\ 15^m$. In B ($16^h\ 27^m$) the principal characteristics of this line are (*a*) a brilliant reversal of the contour corresponding to the full width of the flare filament, (*b*) a great extension of the wings coinciding in position with the narrow central region of the flare, and (*c*) a greater extension for the red wing from the centre of *Hα* absorption than for

* The microphotometer tracing obtained subsequently shows a certain extension to 15·9 A.

† Communicated privately, with the kind permission of the Director of Operational Research, Ministry of Supply. The question of a more general correlation between solar radio noise reception, solar flares and ionospheric fade-outs is discussed by E. V. Appleton and J. S. Hey (*Phil. Mag.*, **37**, 73, 1946.)

‡ *Observatory*, **66**, 347, 1946.

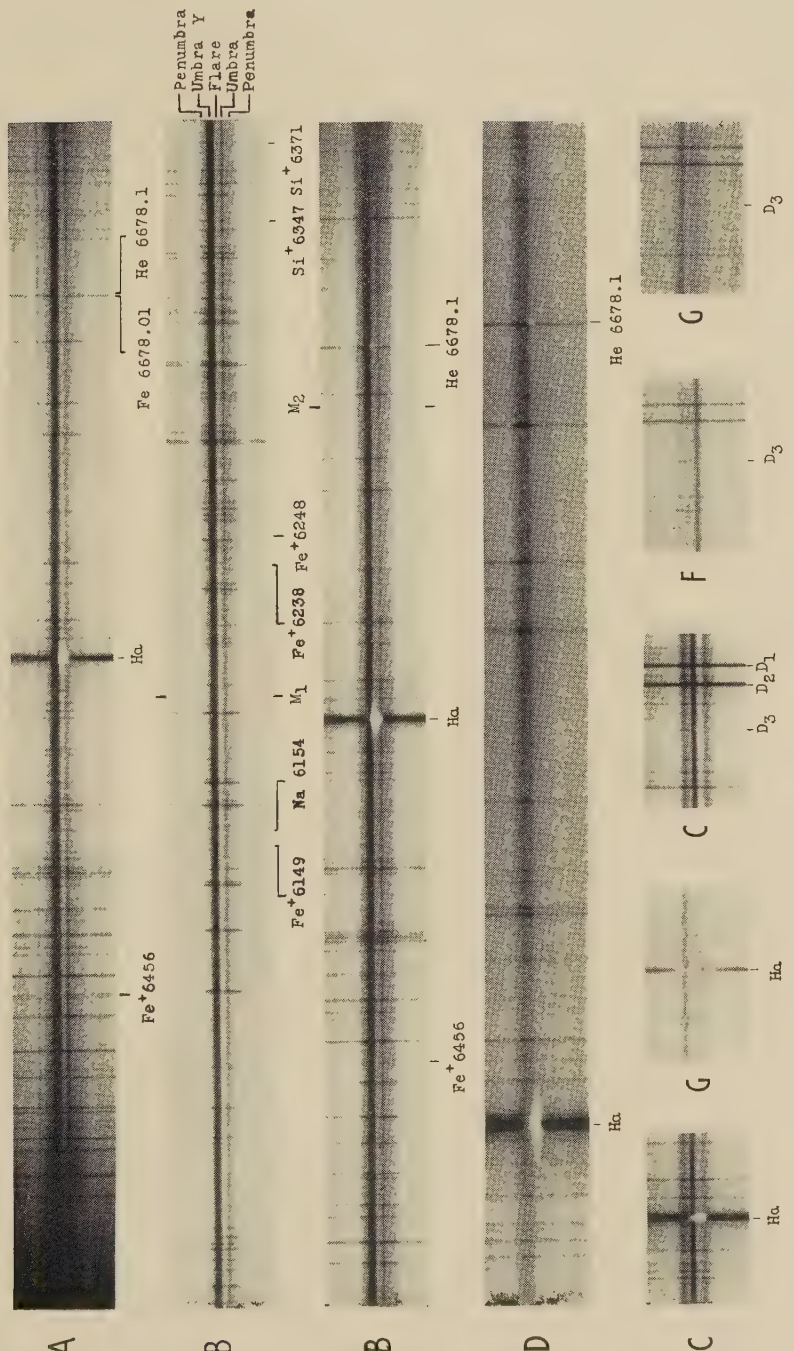
the blue wing (see Section 7). Microphotometer readings give a value for the central intensity of the emission equal to 300 per cent. of the continuous spectrum 15 A. from $H\alpha$. However, since the centre of the emission line was inevitably over-exposed, this reading must be taken as an approximate value only, but as a lower limit to the true intensity. In spectrum G (Plate 13, No. 2 of bottom row) the $H\alpha$ emission can be seen to have split up and to have spread over a greater area, and the microphotometer shows that the intensity was at this time ($17^h\ 15^m$) just equal to the continuous spectrum outside the spot, that is to say, 48 minutes after the time of maximum brightness.

These remarkable appearances of the $H\alpha$ line should be compared with some of Lockyer's drawings and references to what he calls "bright lozenges" superimposed upon the hydrogen absorption lines, which he observed during the month

TABLE II

*Lines affected in Spectrum B*

| Revised Rowland Table λ | Identification | Intensity on Disk | Excitation Potential | Confirmation | Remarks |
|---|---|---|---|---|---|
| 6772·33 | *Ni* | 2 | 3·64 | ... | Filled in. |
| 6678·15 | *He* | ... | 21·13 | A, D, E | >continuous spectrum : 1·1 A. wide. |
| 6643·65 | *Ni* | 5 | 1·67 | D | Partially filled in. |
| 6633·77 | *Fe* | 2 | ... | ... | |
| 6609·13 | *Fe* | 3 | 2·55 | ... | |
| 6593·89 | *Fe* | 4 | 2·42 | ... | |
| 6592·93 | *Fe* | 6 | 2·72 | ... | |
| 6569·23 | *Fe* | 5 | ... | ... | May be due to wing of *Hα* emission. |
| 6562·82 | *Hα* | 40 | 10·15 | A, B, C, D, E and G | ≫continuous spectrum : 15 A. wide. |
| 6496·92 | *Ba*$^+$ | 4 | 0·60 | ... | Partially filled in. |
| 6496·48 | *Fe* | 2 | 4·77 | ... | |
| 6495·75 | *Fe* ? | 1 | 4·81 | ... | |
| 6495·00 | *Fe* | 8 | 2·39 | ... | |
| 6493·80 | *Ca* | 6 | 2·51 | ... | |
| 6471·68 | *Ca* | 5 | 2·51 | ... | |
| 6456·40 | *Fe*$^+$ | 3 | 3·89 | A, D | >continuous spectrum. |
| 6400·02 | *Fe* | 8 | 3·59 | ... | Partially filled in. |
| 6400·33 | *Fe* | 2 | 0·91 | ... | |
| 6393·62 | *Fe* | 7 | 2·42 | ... | |
| 6371·36 | *Si*$^+$ | 1 | 8·09 | D | >continuous spectrum : blended with faint ***Fe***. |
| 6347·10 | *Si*$^+$ | 2 | 8·09 | D | >continuous spectrum. |
| 6247·57 | *Fe*$^+$ | 2 | 3·87 | ... | Partially filled in. |
| 6238·40 | *Fe*$^+$ | 2 | ? | ... | Completely filled in. |
| 6154·26 | *Na* | 2 | 3 | ... | |
| 6149·25 | *Fe*$^+$ | 2 | ? | ... | |
| 5875·79 | *He*($D_3$) | ... | 20·87 | C, F and G | In strong emission and absorption (C) |




Spectra of solar flare 1946 July 25.

of April, 1870 (see *Solar Physics*, p. 402). Who can doubt that we have here photographs of a similar phenomenon to that which Lockyer so aptly described during the great sunspot maximum of 76 years ago?

*Helium.*—The most interesting feature here is the appearance of $\lambda 6678{\cdot}1$ in emission on plates A, B, D and E, covering the period from $16^h\ 25^m$ to $16^h\ 50^m$. There is no trace of it at $17^h\ 15^m$. On plate B the emission is 1·1 A. wide and is blended with the iron absorption line $\lambda 6678{\cdot}01$. Richardson and Minkowski* report having observed this line in emission in a flare caught on the limb, but state that it was not found in any flares seen upon the disk. They point out that just as the emission of $H\alpha$ is followed by the emission of $L\alpha$, so the emission of helium $\lambda 6678\ (2^1P - 3^1D)$ will be succeeded by that of $\lambda 584\ (1^1S - 2^1P)$. The latter line is capable of ionizing directly all constituents of the Earth's atmosphere, helium excepted. Helium $\lambda 5876\ (D_3)$ appears in emission on C (Plate 13, No. 3, bottom), the earliest ($16^h\ 35^m$) exposure covering this region of the spectrum. The emission extends to the full width of the flare filament, giving way to absorption over the spot penumbra north and south of the flare itself. The absorption can also be traced beyond the southern boundary of the spot for about 30,000 km. out over the photosphere. In plates F ($16^h\ 52^m$) and G ($17^h\ 15^m$) the emission of this line is no longer visible, though the absorption is still conspicuous over the same area. Richardson and Minkowski have reported the appearance of $\lambda 5876$ in emission in a flare occurring at the limb, but in disk flares it was not detected either in emission or absorption. Allen†, on the other hand, records both $\lambda 6678$ and $\lambda 5876$ in his list of emission lines seen in flare spectra, presumably on the disk.

In spectrum G ($17^h\ 15^m$) we notice a dark "blob" projecting outwards on the red side of the $\lambda 5876$ absorption line. This coincides precisely in position with a similar sideways extension of the $H\alpha$ line on the same plate (*cf.* Nos. 2 and 5, bottom of Plate 13). These are the spectral images in helium and hydrogen light respectively of an absorption flocculus which was observed visually with the spectrohelioscope at about the same time. Measures of these red-ward displacements with a travelling microscope, translated into terms of sightline velocity, give values of +93 km./sec. and +100 km./sec. for helium and hydrogen respectively. The difference, in view of the diffuse nature of the images, is not to be interpreted as having any real significance. The same descending flocculus was followed visually with the spectrohelioscope from $17^h\ 30^m$ onwards (Fig. 1c), and at $17^h\ 50^m$ its sightline velocity, determined with the line-shifter, was found to be +75 km./sec. Its velocity was again measured at $18^h\ 05^m$, giving a value of +70 km./sec.

*Sodium.*—No spectrum of the D lines is available before C ($16^h\ 35^m$) in which there is observable possibly a very slight filling-in of $D_1$ and $D_2$, nothing more. This is curious, since H. W. Newton‡, in his summary of early visual observations of flare reversals, draws attention to the number of occasions on which the D lines were seen brightly reversed over what were evidently flare filaments. Maunder, for example, reported $D_1$ and $D_2$ greatly broadened (1882 November 18), each line "appearing as a very broad and ill-defined dark band about 6 A. in width, with

* *Ap. J.*, **89**, 352, 1939.
† *M.N.*, **100**, 639, 1940.
‡ *M.N.*, **103**, 250, 1943.

a sharp, narrow emission line in the centre". Allen also records* $D_1$ and $D_2$ reversed, but they are not listed as affected by Richardson and Minkowski.†

*Silicon.*—The line at λ6347·1 of ionized silicon is brightly and sharply reversed in B ($16^h\ 27^m$) and is still visible faintly in D ($16^h\ 37^m$). λ6371·4, also an ionized line, is reversed as a wide and diffuse line in B. Neither of these reversals have been previously reported.

6. *Continuous spectrum.*—On plate B, which we have seen was exposed probably within one minute of the peak intensity of the flare, there is to be found a narrow dark streak (bright in the positive of Plate 13, second spectrum) becoming visible at about λ6450 and gradually increasing in density with shorter wave-lengths, until the end of the plate is reached at λ6000. There is no trace of this in the longer wave-lengths (λ6800) of the same plate. It coincides precisely in position with the most intense part of the flare filament and also with the extended wings of the *Hα* emission, though it is clearly not an extension of these wings. The reality of this marking is placed beyond doubt by a comparison of the microphotometer tracings made across the plate at $M_1$ (λ6202) and at $M_2$ (λ6661) in a direction *transverse* to the plane dispersion. Tracing (*c*) of Fig. 3, shows the sharp peak (P) corresponding to this narrow streak running along the spectrum. All the other features except P can be found in the tracing (*d*) across the red end of the same plate, where we have in the same position only the intensity level of the penumbra between the two umbrae (the scale of the two tracings is different because of the change in plate sensitivity between the two wave-lengths). We evidently have here an enhancement of the continuous spectrum (of the order of 10 per cent.) over the penumbra, occurring during the very short period of the radiation burst, which, if it extended throughout the length of the visible region of the spectrum, would have been sufficient to render visible by integrated light that part of the flare filament overlying the central part of the sunspot. It is well known, of course, that such an observation was made upon one occasion previously. This was the phenomenon of 1859 September 1, which, by reason of its characteristic geomagnetic effects, has been recognized‡ as the earliest, and possibly the greatest, of recorded flares. This became visible for a few minutes in integrated light and was seen independently by Carrington and Hodgson as a pair of brilliant patches over the giant sunspot, both observers using ordinary telescopic means.

Richardson and Minkowski have also described what may have been a similar effect. For, although they state that "None of the photographs shows any emission throughout the continuous spectrum over the eruptive (i.e. flare) flocculi", yet they mention that "A bright streak appears in the visual region (no exposures were made in the ultraviolet), on photographs of the eruption (flare) of 1938 December 7, but *not* at the position of the line emission".§

It is evident that this is a phenomenon of short duration, occurring only during the radiation burst, and that further observations will require the taking of many spectra (not necessarily of high dispersion) at intervals of about one minute during the early stages of flare activity.

7. *Contour of the Hα reversal.*—In a former communication‖ it was shown that

* *Loc. cit.*
† *Loc. cit.*
‡ *M.N.*, **103**, 244, 1943 ; *J.B.A.A.*, **53**, 244, 1943.
§ *Loc. cit.*
‖ *Loc. cit.*

the mean wave-length of the extinction points of the wings of the bright $H\alpha$ reversal exhibited a shift to the red, relative to the centre of $H\alpha$ absorption. Measurements made with the line-shifter of the spectrohelioscope upon 22 flares gave a very consistent value of about 0·3 A. to the red for the brightest flares and lesser values for those of Classes 2 and 1. It was not possible at the time to say whether this was caused by a shift of the emission line as a whole, or was due to an asymmetry of the contour, though evidence was put forward to show that it could not be attributed to sightline velocity of the flare filaments. A similar series of measurements made during the flare of July 25 once again shows that the mean wave-length of the wing extremities is displaced about 0·3 A. to the red. The points on the graph (Fig. 2B),

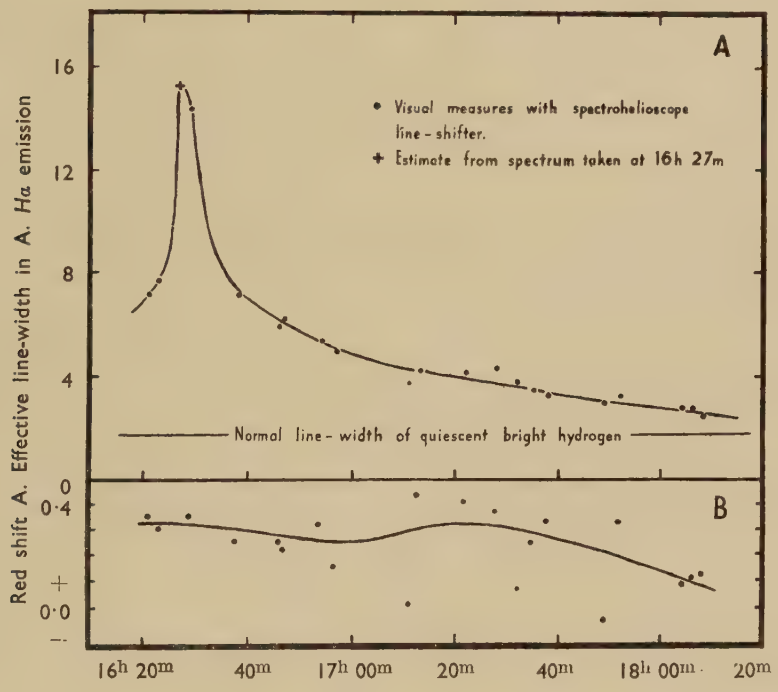



FIG. 2.

*Graph A illustrates the course of the flare, as determined by measures of effective line-width of the bright Hα emission. The extremities of the bright wings of the line were obtained by the method described in a previous paper* (*M.N.*, **103**, 3, 1943).

*Graph B. In the Hα emission of a flare the red wing extends further from the centre of Hα absorption than the blue wing. In B above, the mean wave-length of the extremities of the bright emission is plotted as a displacement from the centre of Hα absorption* (λ 6562·8). *Similar results obtained in connection with 22 earlier flares will be found in M.N.*, **103**, 3, 1943.

illustrating the red shift against time, show a good deal of scatter, but only one of the twenty measures is negative. In an endeavour to corroborate the evidence of the line-shifter, a microphotometer tracing (Fig. 3*a*) was made along the $H\alpha$ emission of plate B. Though the scale was small and the central intensity much over-exposed, the tracing shows quite certainly an extension of 8·3 A. to the red and of 7·6 A. to the blue. These give a mean wave-length 0·35 A. greater than that of the centre of $H\alpha$ absorption, in good agreement with the line-shifter results. It is not possible to say whether this effect is due to the existence of an absorption

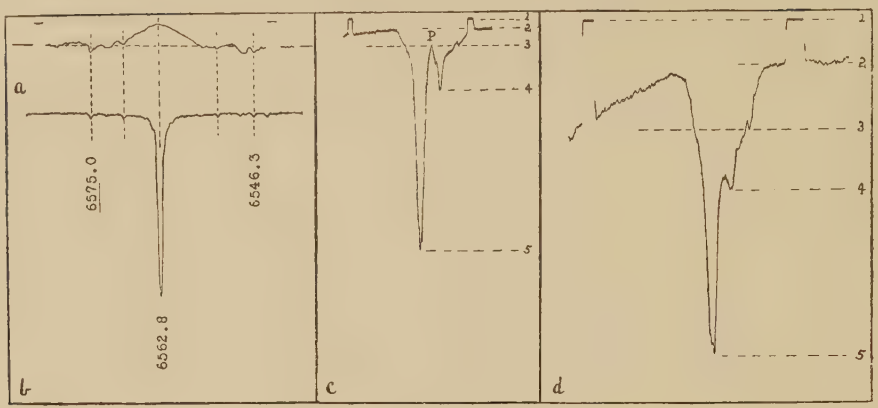



FIG. 3.—*Microphotometer tracings.*

*a.—Tracing along the bright emission of Hα in spectrum B (Plate 13). Red to left.*

*b.—Tracing along the continuous spectrum of the photosphere adjacent to a.*

*c.—Tracing across spectrum B (Plate 13) at $M_1$, in a direction transverse to the plane of dispersion* (λ 6202).

*d.—A similar tracing at $M_2$* (λ 6661). *The various levels are*: 1, *density* $= \infty$ ; 2, *intensity of adjacent continuous spectrum of photosphere*; 3, *enhanced continuous spectrum of flare* (*P*) *at* λ 6202, *but not appreciable at* λ 6661; 4, *continuous spectrum of small umbra*; 5, *continuous spectrum of large umbra Y* (*see Fig.* 1).

band on the short-wave side of the emission (as previously suggested), for the small width of the emission spectrum made it impossible to use the microphotometer at its full sensitivity.

In conclusion, I should like to acknowledge my indebtedness to Professor R. W. Ditchburn for kindly allowing me to use the Moll Recording Microphotometer at Trinity College, Dublin, to Mr T. H. Mason for the excellence of the spectrum enlargements and to the Government Grant Committee of the Royal Society for further assistance in connection with this work.

*Sherborne, Dorset* :
1946 *September* 24.

# THE ORIGIN OF THE NIGHT SKY LIGHT

*D. R. Bates*

(Communicated by Professor H. S. W. Massey)

(Received 1946 November 18)

*Summary*

Theories of the origin of the night sky light are discussed. It is pointed out that evidence from the absolute intensity, the altitude, the space and time variations, the correlations with magnetic activity, and the particular lines and bands appearing in the spectrum, indicates that the emission is not solely connected with the potential energy associated with the region of dissociated oxygen or with the ionized layers but partly arises from incident charged particles (such as produce aurorae). The suggestion is made that it is the action of the Störmer current that allows these particles to approach sufficiently close to the Earth at low latitudes.

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The night sky spectrum consists mainly of the auroral green and nebular red lines of atomic oxygen, and the Vegard-Kaplan, Lyman-Birge-Hopfield and First Positive bands of molecular nitrogen.* Various attempts have been made to provide an explanation for these emissions by treating them as connected with some source of potential energy within the Earth's atmosphere. The only such sources that appear to merit discussion are:—

(*a*) the region of dissociated oxygen (*cf.* S. Chapman †);

(*b*) the ionized layers (*cf.* S. K. Mitra and S. N. Ghosh ‡).

Chapman has considered (*a*) in some detail. He suggested that two oxygen atoms on combining to form an oxygen molecule might give up their excess energy to excite some third body which could then radiate,

$$O+O+X\rightarrow O_2+X', \tag{1}$$

$$X'\rightarrow X+h\nu. \tag{2}$$

The energy available in (1) is sufficient for the production of the oxygen lines but it is not sufficient for the production of the nitrogen bands. For these Chapman found it necessary to assume that additional energy is provided by one of the colliding oxygen atoms being in an excited state. Bates and Massey § however have recently examined this hypothesis and have shown that the resulting intensity of the bands would in fact be negligibly small.

Chapman has also considered (*b*). It would seem that this cannot account for the main atomic oxygen radiation as the number of parent particles is too low to yield the number of photons that have been found by Rayleigh ‖ to be emitted during a night. The position regarding the nitrogen bands appeared at first less certain. As particular interest is naturally attached to these because of the failure of the alternative theory already mentioned, Bates and Massey investigated the

* *Cf.* C. T. Elvey, P. Swings and W. Linke, *Ap. J.*, **93**, 337, 1941. The identifications are still subject to revision.

† *Proc. Roy. Soc.*, A, **132**, 353, 1931; and *Phil. Mag.*, **23**, 657, 1937.

‡ *Sci. and Cult.*, **9**, 46, 1943; and *Proc. Nat. Inst. Sci.* (*India*), **9**, 301, 1943.

§ *Proc. Roy. Soc.*, A, **187**, 261, 1946.

‖ *Proc. Roy. Soc.*, A, **129**, 458, 1930.

possibilities. From a study of the spectroscopic data given by various workers, they judged the intensity involved to be considerably less than that of the oxygen lines—such a procedure was, of course, open to criticism but it seemed to be the best that could be done at the time. Accepting the rough estimation provided, they found that the transfer action

$$N_2^+ + O^- \rightarrow N_2' + O' \qquad (3)$$

might *just* yield sufficient excited nitrogen molecules. This conclusion however depends on the correctness of the ionic recombination theory of the equilibrium of the ionized layers which, as Bates and Massey emphasized, is beset with difficulties. A new theory of the equilibrium of the ionized layers has now been developed. An account of this will be published elsewhere and it need only be mentioned here that it invokes mainly such dissociative recombination reactions as

$$O_2^+ + e \rightarrow O' + O', \qquad (4)$$

or

$$NO^+ + e \rightarrow N' + O', \qquad (5)$$

which would yield only atomic lines. The process

$$N_2^+ + e \rightarrow N_2' + h\nu \qquad (6)$$

is probably very slow.

However, the whole situation is in any case altered by some important measurements by Garrigue* which, owing to wartime conditions, were not available earlier. Using photometric methods, he determined the relative intensities of the blue, the green and the red regions of the spectrum of the night sky at Pic du Midi. His results are summarized below.

| Spectral region | Blue | Green | Red |
|---|---|---|---|
| Relative intensity † | 1 | 1·4 | 1·8 |
| (Mean of Aug. and Oct. 1939) | | | |

Since the nitrogen bands contribute largely to the blue region, it would seem that their integrated intensity must be comparable with that of the oxygen lines. If this is the case, Chapman's argument regarding the number of parent particles can at once be used to dismiss the ionized layers as a possible origin.‡

It appears then from the above that the oxygen lines may be due to (*a*) but not to (*b*), and that the nitrogen bands are extremely unlikely to arise from (*b*) and certainly cannot from (*a*). The position is therefore very unsatisfactory. If now we turn to the observational data which is gradually being accumulated on the general properties of the night sky radiation, we will see that the evidence strongly indicates that both (*a*) and (*b*) must be abandoned as the sole source and that instead we shoul dattribute part of the emission to the action (direct or indirect) of

(*c*) incident charged particles (such as produce the aurorae)

as suggested by Dauvillier§ and others.

* *C.R. Acad. Sci., Paris*, **209**, 769, 1939.

† The scale is such that sunlight has equal values in the three regions.

‡ To maintain a steady emission throughout the night approximately $10^{14}$ parent particles/cm.$^2$ column are needed. The number of electrons present in the ionized layers at midnight is only of order $10^{12}$/cm.$^2$ column and the number of ions cannot be much greater.

§ *Rev. gén. Elect.*, **31**, 303, 443, 477, 1932.

We consider the data under various heads.

(i) *Altitude.*—Several early workers, notably Cabannes and Dufay * and Garrigue †, by determining the variation of the intensity with zenith distance, concluded that the height of the luminous region must be very great. The most recent investigations are those of Elvey and Farnsworth ‡ and of Abadie, Vassy and Vassy. § The radiations studied were $\lambda 5577$, $\lambda 6300$ (of atomic oxygen) and $\lambda 6560$ (of unknown origin). Elvey and Farnsworth found the source in all cases to be a height of 500 km. Although Abadie, Vassy and Vassy confirmed the emission from immense heights (which they gave as 900–1000 km.), they suggested that in addition there was a rather less intense emission from much lower (perhaps about 70 km.). Now Bates and Massey || have shown that (*a*) can only give rise to a thin luminous layer situated at about 100 km. It is clear also that most of the emission from (*b*) occurs in the region of 300 km. (the height of the F-layer). These sources by themselves are therefore not reconcilable with the zenith angle measurements—though it is possible that (*a*) might account for the lower layer of Abadie, Vassy and Vassy.

Further experimental work is very desirable, particularly regarding the location of the source responsible for the various nitrogen bands.

(ii) *Space variations.*—It is well known that the night sky is not equally bright in all directions and, instead, appears to show patches of luminescence. Such an effect would not be expected with (*a*), and even with (*b*) it is doubtful if it could be pronounced.

The dependence on latitude is also of great significance. Rayleigh and Jones ¶ and Garrigue ** have found evidence that the brightness of the night sky increases on going away from the Equator. This is confirmed by the investigations of Fessenkoff †† whose results are given in the following table.

| Latitude (N.) | Brightness of night sky ($10^{-3}$ stilb |
|---|---|
| 44°·4 | 0·30 |
| 46·8 | 0·31 |
| 55·8 | 0·57 |
| 80·3 | 1·20 |

The general trend is very striking and once again is not easily explained with either (*a*) or (*b*).

(iii) *Time variations.*—The intensity of the night sky emission is subject to remarkable fluctuations. Thus Rayleigh ‡‡ has observed occasions on which it differs by a factor of as much as eight—and of course this is probably not the extreme range. This in itself is very suggestive, but even more important is the fact that Rayleigh and Jones §§ find a close correlation between the fluctuations of the blue, the green and the red spectral regions, giving a strong presumption that the main emissions have a common origin, whereas in the potential energy source

* *C.R. Acad. Sci., Paris*, **198**, 306, 1934.
† *C.R. Acad. Sci., Paris*, **202**, 1807, 1936.
‡ *Ap. J.*, **96**, 451, 1942.
§ *C.R. Acad. Sci., Paris*, **218**, 164, 1944.
|| *Loc. cit.*
¶ *Proc. Roy. Soc.*, A, **151**, 22, 1935.
** *C.R. Acad. Sci., Paris*, **209**, 769, 1939.
†† *C.R. Acad. Sci., U.R.S.S.*, **32**, 320, 1941.
‡‡ *Proc. Roy. Soc.*, A, **131**, 276, 1931.
§§ *Loc. cit.*

theories (*a*) has to be invoked to account for the oxygen lines and (very dubiously) (*b*) for the nitrogen bands.

The emission is maintained throughout the dark hours without marked diminution. Apparently indeed it may reach a maximum near midnight: this has been observed in the case of $\lambda$ 5577 by McLennan *, and in the case of the blue region of the spectrum (nitrogen bands) by Rayleigh.† Though many other workers ‡ have confirmed the effect, there is evidence that it may not always occur.§ (*a*) can certainly yield a sustained emission but it is not obvious that it can give rise to the maximum. It is clear too that (*b*) is once again in serious difficulties, since the radio measurements ‖ show directly that in general the peak electron density in the ionized layers decreases to a very low minimum shortly before dawn.

The seasonal variations have been investigated by Rayleigh and Jones ¶, Cabannes and Dufay **, Elvey, Swings and Linke †† and others. It seems that the intensity of the major emissions tends to be greatest at the equinoxes. This is not readily explained by (*a*) but it has been pointed out by Martyn and Pulley ‡‡ that the same general variation is shown by the F-layer. In view, however, of the utter failure of (*b*) in other respects, little weight can be given to the agreement—though it may be that the fundamental cause of the night sky variation and of the F-layer variation is the same.

Rayleigh and Jones §§ have found the intensity in the various regions to increase with sunspot number, but so many upper atmospheric phenomena show this effect that it is not a very useful criterion in the present connection—though it may be remarked that the change in the degree of ionization in the night F-layer appears too great.

It will be noted from the above that, as far as has been observed, the principal radiations have similar time variations.|||| This supports the evidence from the fluctuations that they have a common origin. That the origin is, in fact, (*c*) is suggested by the close resemblance of each variation to the corresponding curve showing the frequency of aurorae occurrences throughout the day, the year and the sunspot cycle.¶¶

(iv) *Correlations with magnetic activity.*—Some indication of a correlation of enhanced night sky emission with magnetic activity was shown by the work of

* *Proc. Roy. Soc.*, A, **120**, 327, 1928.

† *Proc. Roy. Soc.*, A, **124**, 395, 1929.

‡ *Cf.* N. Dobrotin, I. Franck and P. Čerenkov, *C.R. Acad. Sci., U.R.S.S.*, **1**, 110, 1935; N. Wassmuth, V. Werzner, S. Tibilov and S. Freivert, *C.R. Acad. Sci., U.R.S.S.*, **19**, 5, 405, 1938; N. E. Bradbury and W. T. Sumerlin, *Terr. Magn. Atmos. Elect.*, **45**, 19, 1940; and C. T. Elvey, P. Swings and W. Linke (*loc. cit.*).

§ *Cf.* J. V. Karandikar, *Indian J. Phys.*, **8**, 547, 1934; C. T. Elvey and A. H. Farnsworth, *loc. cit.*

‖ *Cf.* L. V. Berkner and E. O. Hulburt, *Terrestrial Magnetism and Electricity* (Ed. J. A. Fleming), New York, 1939.

¶ *Loc. cit.*

** *C.R. Acad. Sci., Paris*, **200**, 878, 1935.

†† *Loc. cit.*

‡‡ *Proc. Roy. Soc.*, A, **154**, 455, 1936. This is an oversimplified statement. J. Bannon and F. W. Wood (*Terr. Magn. Atmos. Elect.*, **51**, 89, 1946) investigated the behaviour of the nocturnal ionization for a considerable number of stations. They found that it differs from place to place.

§§ *Loc. cit.*

|||| An exception will be mentioned later.

¶¶ For summary of data on aurorae *cf.* L. Vegard, *Terrestrial Magnetism and Electricity* (Ed. J. A. Fleming), New York, 1939.

Rayleigh and Jones and the recent researches of Barber* appear to have fully established a connection. It is doubtful if (*a*) can account for this as it seems unlikely that the great reservoir of dissociated oxygen is appreciably augmented during disturbances. The ionized layers are certainly affected† but the changes that take place are not such as would obviously lead to an increased emission.

It is perhaps worth mentioning here that, judging from the extent of the association between the occurrence of auroral and magnetic activity ‡, (*c*) does not necessitate the above correlation to be exact.

(v) *Spectra.*—There is a considerable difference between the spectrum of the night sky and that of aurorae, but as Déjardin§ has pointed out, it lies principally in the degree of the excitation, which is lower for the former. In view of the large number of possibilities arising from the miscellaneous actions (1)–(6) (and from various actions arising with other parent particles) it would scarcely be anticipated that so many of the main lines and bands in the night sky, if produced by (*a*) and (*b*), would simply be those occurring in aurorae. It seems perhaps more natural to assume that the two emissions arise in a similar manner.‖ However the reason for the low excitation energy in the night sky spectrum is obscure.

The survey we have given shows that (*a*) and (*b*) encounter many difficulties. Though some of these may indeed not be insuperable, the evidence as a whole is such as to leave little doubt that the entire night sky light originates from neither source. If (*c*) is accepted, however, all the effects described are immediately explicable. Admittedly it is not clear how the charged particles reach low latitudes despite the Earth's magnetic field: but this is not surprising, since even aurorae are still not properly understood. We would like to put forward as a tentative suggestion that the action of Störmer's proposed equatorial current ¶ is responsible. Evidence in favour of this is provided by Brüche's celebrated experiments** illustrating the motion of electrons near various magnetic models of the Earth. These show (in agreement with calculation) that in the field of a simple dipole, the electrons only strike the Earth at very high latitudes (well above the auroral zone) and that at lower latitudes they cannot approach closer than some thousands of kilometres. The Störmer current was postulated as a means of removing the discrepancy regarding the position of aurorae. Brüche was able to verify the possibility of this. Inspection of his published photographs†† indicates that, in addition, *at low altitudes it becomes permissible for a relatively small number of electrons to approach the Earth within* 500–1000 *km.*: our hypothesis is that these give rise to part of the night sky light. It is of course necessary for confirmatory experiments or calculations to be performed.

* *Nature*, **148**, 88, 1941.

† *Cf.* L. V. Berkner, *loc. cit.*

‡ *Cf.* S. Chapman and J. Bartels, *Geomagnetism*, Oxford, 1940.

§ *Rev. mod. Phys.*, **8**, 1, 1936.

‖ This cannot however be regarded as conclusive evidence in favour of (*c*). Thus S. K. Mitra (*Nature*, **155**, 786, 1945 ; and **157**, 692, 1946) supports (*b*) as the source of the night sky light and accepts incident charged particles as the cause of aurorae but suggests that (3) is the main process finally responsible for the emission in both cases.

¶ *Arch. Sci. phys. nat.*, **32**, 1911 ; and **33**, 1912.

** *Terr. Magn. atmos. Elect.*, **36**, 41, 1931.

†† These are reproduced by S. Chapman and J. Bartels in *Geomagnetism* and by J. Bartels in *Terrestrial Magnetism and Electricity*.

We thus suggest incident charged particles contribute to the main emissions (in particular those common to aurorae). It is to be emphasized, however, that we do not suggest that they produce all the emissions. There is, for example, no doubt that the D lines of sodium are due to some special mechanism.* It is conceivable also that (*b*) may cause the after-sunset enhancement of the red line of oxygen that has been observed by Elvey, Swings and Linke † and others. The many very non-auroral sky lines or bands have probably to be considered separately: Clearly as much information as possible on the properties (i)–(iv) of individual lines and bands must be obtained.‡

In conclusion, I should like to thank Professor Massey for many helpful discussions.

*University College,*
*London :*
1946 *November.*

* *Cf.* S. Chapman, *Ap. J.*, **90**, 309, 1939 ; and D. R. Bates, *Terr. Magn. Atmos. Elect.*, **52**, 71, 1947.

† *Loc. cit.*

‡ In particular it would be helpful to study the red lines of oxygen both when the residual effect is pronounced and when it is negligible.

# ON THE RELATIONS BETWEEN IONOSPHERE, SUNSPOTS AND SOLAR CORONA

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(Communicated by the Astronomer Royal)

(Received 1946 December 27)

*Summary*

Analysis of the ultra-violet solar radiation producing the ionospheric layers E, $F_1$ and $F_2$, particularly its correlation to sunspots and coronal intensities in $\lambda$ 5303, shows that for wave-lengths $\lambda < 700$ A. the inner corona is about $10^5$ times brighter than the photosphere and that its total intensity varies during a solar cycle by a factor of the magnitude 10. The variations of intensity, which are superposed on a constant background-brightness (permanent corona) arise from extended, long-lived bright coronal patches, whose total intensity is controlled, with a certain delay, by the underlying spot groups. The distribution of brightness within these regions corresponds roughly to the reverse of the one observed in $\lambda$ 5303.

With increasing wave-length, that is approaching the end of the Lyman-continuum ($\lambda \sim 800$–$912$), the intensity of the corona reverts to that of the chromospheric faculae, whose intensity, apart from a negligible limb darkening, follows the development of the spot group under it with a delay of a few days.

At $L_\alpha$($\lambda$ 1215) the chromospheric faculae and particularly the eruptions once again define the total intensity of the Sun.

1. *The laws of the ultra-violet radiation, producing the ionospheric layers.*—The main difficulty in correlating phenomena in the upper terrestrial atmosphere to solar phenomena is the close relationship between different solar phenomena. Since the terrestrial effects to be considered here are nearly all produced by radiation or particles which cannot be measured directly by the observer on the Earth's surface, we are unable to localize directly the radiation sources on the Sun. The ensuing difficulties can be partly overcome by the use of statistical methods.

As has been shown in a former paper *, the absolute extraterrestrial intensities which are necessary to maintain the state of ionization in the E-, $F_1$- and $F_2$-layers, can be taken from the variation of critical frequencies during total solar eclipses or during sunrise and sunset. If we add certain suppositions on the nature of the ionization processes and on the chemical composition of the Earth's atmosphere we can say something also about the wave-lengths of the ionizing radiation. According to J. N. Bhar † the ionizing wave-length regions are for the E-layer $\lambda$ 744–661 A., for the $F_1$-layer $\lambda < 661$ A., and for the $F_2$-layer $\lambda$ 910–744 A. The required absolute intensities are far above the photospheric intensity in the corresponding wave-length regions and exceed these in the E- and $F_1$-layer by a factor of about $10^5$ and in the $F_2$-layer by about $10^3$. Even after eliminating an annual period of terrestrial origin and the distorting influence of the varying geomagnetic field,

* *Ann. Astrophys.*, **8**, 210, 1945.

† *Proc. nat. Inst. Sci. India Pap.*, **33**, 363, 1938.

as well as after reduction for perpendicular incidence of the solar radiation, a remarkable fluctuation remains. It can be represented very well as a function of the relative sunspot numbers. If we call $R_3$ the moving daily means of three days and $R_{27}$ the moving daily means of a solar rotation of the sunspot numbers of the total disk we get the following statistical relations for the intensities, composed of three components I, II, III. The units are (10 Mc./sec.)$^3$ for $\mathcal{J}_E$ and (Mc./sec.)$^4$ for $\mathcal{J}_{F_1}$ and $\mathcal{J}_{F_2}$ (*cf.* [I] and [III]).

$$\left.\begin{aligned} \mathcal{J}_E &= 44{\cdot}4 + 0{\cdot}09R_{27} + 0{\cdot}22R_3; \quad (744\text{–}661\text{ A.}), \\ \mathcal{J}_{F_1} &= 444 + 1{\cdot}58R_{27} + 2{\cdot}09R_3; \quad (<661\text{ A.}), \\ \mathcal{J}_{F_2} &= 622 + 4{\cdot}10R_{27} + 15{\cdot}4R_3; \quad (910\text{–}744\text{ A.}). \end{aligned}\right\} \qquad (1)$$

I II III

The component I represents a radiation of constant intensity which is independent of the cycle. The component II follows the spot numbers with a time lag greater than a solar rotation, whilst the component III follows them practically without time lag.

The correlation coefficients $r(\mathcal{J}, R)$ between spot numbers and "ionospheric intensities" give rather high values. In the annual means we get as an average:

$$r(\mathcal{J}_{F_2}, R) = 0{\cdot}989; \qquad r(\mathcal{J}_{F_1}, R) = 0{\cdot}9986; \qquad r(\mathcal{J}_E, R) = 0{\cdot}833.$$

In the rotational means (component II):

$$r(\mathcal{J}_{F_2}, R) = 0{\cdot}529; \qquad r(\mathcal{J}_{F_1}, R) = 0{\cdot}779; \qquad r(\mathcal{J}_E, R) = 0{\cdot}704.$$

In the 3-day means (component III) the coefficients lie between zero and the following maximum values:

$$r(\mathcal{J}_{F_2}, R) \approx 0{\cdot}5; \qquad r(\mathcal{J}_{F_1}, R) \approx 0{\cdot}8; \qquad r(\mathcal{J}_{E_2}, R) \approx 0{\cdot}6.$$

All coefficients show a falling tendency approaching the minimum of solar activity.

The correlation coefficients of the moving three-day means of the ionospheric intensities $\mathcal{J}_E$, $\mathcal{J}_{F_1}$ and $\mathcal{J}_{F_2}$ are very high between themselves; they are greatest between $\mathcal{J}_{F_2}$ and $\mathcal{J}_E$ and smallest between $\mathcal{J}_{F_1}$ and $\mathcal{J}_E$. They seem to be independent of the solar cycle.

2. *The sources of the ultra-violet radiation.*—It could be shown further that the required high absolute intensities in the far ultra-violet can be produced by the outer layers of the chromosphere or by the inner parts of the corona. If one assigns to the corona a temperature of about $10^6$ degrees in accordance with different observations*, the radiation intensity emitted by the free-free and free-bound transitions of the electrons and protons of the corona differs in the region of about 600 A. by more than a factor $10^5$ from the photospheric intensity.

We get for the "Ergiebigkeit" of the coronal matter of free-free and free-bound radiation in units of the black-body radiation ($T_\odot = 5740°$) for the frequency $\nu$†:

$$\eta_\nu = \frac{Ac^2 e \dfrac{h\nu}{k}\left(\dfrac{1}{T_\odot} - \dfrac{1}{T}\right)}{8\pi h\nu^3\sqrt{T}}\left(1 + \frac{G}{A}\frac{e^{\frac{\chi}{kT}}}{T}\right)n_e^2, \qquad (2)$$

* M. Waldmeier, *Mitt. aargau. naturf. Ges.*, **12**, 185, 1945.
† *Ann. Astrophys.*, **8**, 210, 1945.

where

$$A=\frac{2^2\pi^3}{(6\pi)^{\frac{3}{2}}}\left(\frac{m}{k}\right)^{\frac{1}{2}}\frac{\epsilon^6}{m^2c^3h}=9{\cdot}0\times10^{-12},$$

$$G=\frac{2^9\pi^5}{(6\pi)^{\frac{3}{2}}}\left(\frac{m}{k}\right)^{\frac{3}{2}}\frac{\epsilon^{10}}{m^2c^3h^3}=3{\cdot}3\times10^{-6}$$

are constants, $T$ the temperature, $n_e$ the density of electrons or protons and $\chi$ the energy of ionization from the ground level. The total intensity of the corona, again in units of the photospheric radiation, becomes then

$$\mathcal{J}_\nu=\int_{r_\odot}^{\infty}\eta_\nu(r)dr. \tag{3}$$

We found in this way numerically that the coronal radiation for $\lambda\geqslant700$ A. does not contribute any more to the formation of the ionospheric layers. The influence of the corona is confined mainly to the $F_1$- ($\lambda<661$ A.) and to the E-layers ($\lambda$ 744–661 A.).

Before combining the coronal observations in the visible part of the spectrum with the ionospheric intensities a few remarks may be made about the contribution of the chromosphere. The absorption coefficients in the region 600–900 A. are very much greater than for the visible continuous radiation, even much greater than for $H_\alpha$. The layers, radiating in the far ultra-violet, therefore, ought to be located still outside the level which can be seen in a $H_\alpha$-spectroheliogram. The increase of temperature in the highest parts of the photospheric faculae which follows from their law of limb darkening * and which gives values of $\Delta T>2000°$, therefore, should effect a considerable strengthening of the emission in the far ultra-violet in accordance with a Kirchhoff-Planck function for the argument $T_\odot+\Delta T$. We come to the same increase of temperature in the chromospheric faculae if, as Unsöld † does, we describe the residual intensity of the faculae in the centre of $H_\alpha$, which is about twice that of the normal chromosphere, by the increase of the Kirchhoff-Planck function in the outer layers.

Faculae radiating like a black body would in consequence give for a rise of temperature $\Delta T$ an increase $\gamma$ of intensity, where

$$\log\gamma=0{\cdot}435\frac{hc}{k\lambda_0T_\odot}\left(1-\frac{T_\odot}{T_\odot+\Delta T}\right). \tag{4}$$

We get for $\lambda$ 700 A. the following values:

| $\Delta T$ | 1000° | 2000° | 3000° | 4000° | 5000° |
|---|---|---|---|---|---|
| $\log\gamma$ | 2·3 | 4·0 | 5·2 | 6·3 | 7·1 |

This considerable intensity comes of course only from the granulae of faculae which make up only a small part, about 10 per cent., of what we call generally faculae regions (*plages faculaires*). According to rough estimates in $H_\alpha$ the total brightness of chromospheric faculae varies about in proportion with the spot number $R$. The total area $F$ of the granulae of faculae becomes then ‡, in units of $\pi r_\odot^2$,

$$F=10^{-4}\times R. \tag{5}$$

* H. Raudenbusch, *A.N.*, **268**, 323, 1939.
† *Physik der Sternatmosphären*, p. 458, Berlin, 1938.
‡ *Ann. Astrophys.*, **8**, 210, 1945.

It can be seen that, e. g. for $R = 50$ and $\Delta T = 3000°$ for $\lambda 700$ A., a total deviation of the solar radiation from the photospheric radiation by a factor of about $10^3$ is produced. This value is not so far from that one needed for the formation of the $F_2$-layer ($\lambda 910$–$744$ A.). It must be considered further that the level responsible for the emission of the faculae in the far ultra-violet must be higher than that for which $\Delta T$ has been estimated and that on the other hand the temperature of the faculae must increase with height if it is to pass over into the temperature of the inner corona. The corona above faculae has a higher temperature than normal, as can be concluded from the appearence of the green and red coronal lines over disturbed chromospheric regions. The given value of $\Delta T \approx 2000° \ldots 3000°$, therefore, ought to represent a lower limit only.

We come thus to the conclusion, in accordance with the statistical results summarized in (1), that the neighbouring faculae become important just in that wave-length region where the emission of the corona does not suffice for the ionosphere. The relatively strong and rapidly variable component III (proportional to $15{\cdot}4R_3$ in the $F_2$-layer) seems to come exclusively from the faculae. Like the $H_\alpha$ intensity of faculae it follows the spot numbers practically without delay and can be represented as a linear function of $R$.

3. *Ionosphere and brightness of the corona in* $\lambda 5303$ A.—The intensity of the ultra-violet emission of the corona depends not only on the density but, according to (2), essentially also on the temperature of the electrons. The density of electrons can be determined fairly accurately from the intensity of the photospheric light scattered by them. Good daily photometric measurements of this light are still very laborious and have been done only by B. Lyot on the Pic du Midi. The temperature of the coronal electrons can be estimated in different ways, e. g. by measuring the Doppler width of the coronal lines *, by the excitation-strength of certain coronal lines or by the degree of diffuseness of the scattered H- and K-lines of the Fraunhofer spectrum.† All methods give about the same temperatures, which all are well above $5 \times 10^5$ degrees, even in those regions where no coronal lines are excited. The appearance of the green line ($\lambda 5303$ A.) requires the existence of *Fe* XIV ions, which need an ionization potential of 355 volts. The green line, therefore, can be excited where the mean energy of electrons surpasses about 300 electron volts. That corresponds to a temperature of about $2 \times 10^6$ degrees. Green coronal regions are consequently hotter, and according to (2) ought to have a weaker ultra-violet emission. If we plot the intensity of the free-free and free-bound emission of the corona, in units of the photospheric radiation as a function of the temperature, we find the result represented in Fig. 1. The intensities $I_\nu$ correspond to an isothermal corona of temperature $T$, for which it was assumed that $\int_{r_\odot}^{\infty} n_e^2(r)dr = 7 \times 10^{28}\,\mathrm{cm.}^{-2}$.‡ We conclude from these curves that the corona gives in the far ultra-violet, so to say, a reversal of the picture observed in $\lambda 5303$.

The coronal observations used here have been made at the two observatories of the Fraunhofer-Institut on the Wendelstein (1840 m.) and Zugspitze (3000 m.).

* M. Waldmeier, *loc. cit.*
† W. Grotrian, *Z. Astrophys.*, **3**, 199, 1931.
‡ *Ann. Astrophys.*, **8**, 236, 1945.

during the years 1943–1945.* Both instruments used are coronographs of Lyot's type, with an aperture of 110 mm. and a focal length of 1650 mm. The brightness of the green coronal line has been estimated visually in the eyepiece of a Zeiss prominence spectroscope at 5° intervals along the limb of the Sun in a scale of 50 steps. Simultaneously, the brightness of the terrestrial scattered light was measured in units of the brightness of the Sun's limb to check the accuracy of the estimated coronal intensities. The observational material of two observatories only 100 km. apart and therefore working under similar climatic conditions has, of course, rather a lot of gaps. In correlating the corona with the ionosphere we therefore employ the "superposed epoch method" which only requires a sufficient number of "selected days" with a strong or weak green corona on the eastern or western limb of the Sun. Unfortunately, the corona can be observed only on the limb of the Sun, but its influence on the ionosphere will be greatest when it passes the central meridian of the Sun's disk. We extrapolate the value of coronal intensity on the central meridian from the measurements made about 7 days before on the eastern or about 7 days later on the western limb, remembering that the intensity distribution within the corona shows little variation with time.

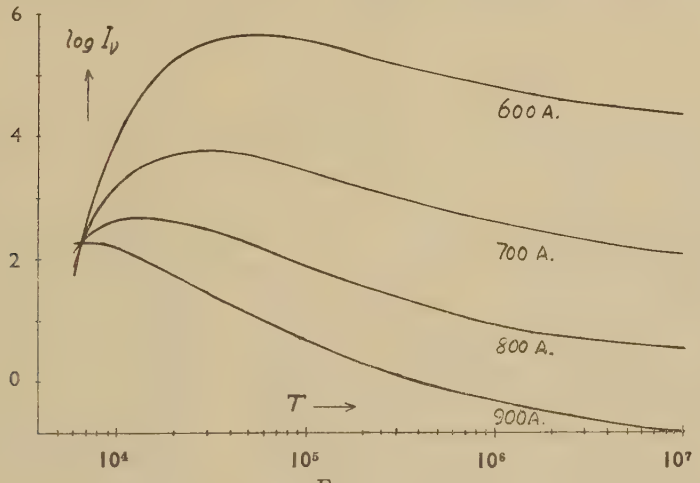


FIG. 1.

As measure of brightness we take simply the sum of all 36 intensities measured along the eastern or western limb respectively (always counting from the north pole of the Sun). When the intensity on the central meridian could be determined from both the eastern and western limbs the mean of both values has been taken.

In Fig. 2 (Plate 14) the resultant coronal intensities $K_Z$ on the central meridian are represented for the solar rotations 1502 to 1550 (1943 January 24 to 1946 October 7) in rows of 27 days. Empty fields correspond to days without observation. The same figure gives also in symbols a representation of the relative spot numbers $R$ of the whole disk and of a small central zone ($r = \frac{1}{4} r_\odot$) $R_{ZZ}$ (observed at the stations of the Fraunhofer-Institut), the daily sums of the "*Dreistündige Potsdamer erdmagnetische Kennziffern*" $\Sigma K_1$, and the daily deviations of the ionospheric ultra-violet intensities from the rotational means $\Delta \mathcal{J}_E$ and $\Delta \mathcal{J}_{F_2}$, all for the rotations 1483 to 1528 (1941 September 29 to 1945 January 21).

* The observations have been made nearly exclusively by H. Haffner and A. Bruzek.

If now we study in the neighbourhood of selected days with a strong corona on the central meridian, e. g. $K_Z > 25$, the behaviour of the ionospheric intensities, we observe in all 3 layers a more or less pronounced depression of the intensity on the day of the passage of the bright corona through the central meridian (0 day). The result is shown in Fig. 3. In spite of the small size of the contributing

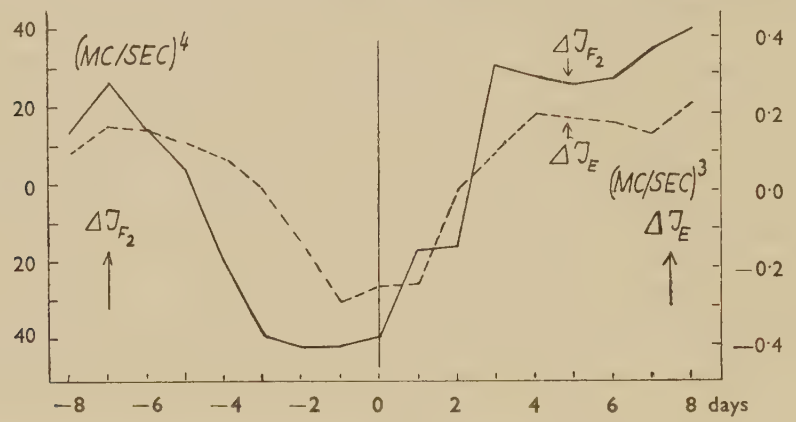


Fig. 3.

coronal area, which is smaller than 10 per cent. of the visible solar disk, there is a sensible diminution of intensity. The regions where the green coronal line appears must be, therefore, much darker than regions where $\lambda$ 5303 is not excited, in accordance with theoretical expectations. The daily deviations of intensity from the rotational mean are given for $\Delta \mathcal{J}_E$ in (Mega-cycles)$^3$, for $\Delta \mathcal{J}_{F_2}$ in (Mega-cycles)$^4$. The corresponding variations of the critical frequencies of the E- and $F_2$-layers can easily be derived. If we correlate the variation of the coronal brightness with that of the ultra-violet intensities, it appears that the negative correlation between them becomes the more pronounced the greater we choose the coronal area for computing the coronal intensity. It follows from this that the limb darkening of the coronal ultra-violet radiation must be very small or that the regions of the corona near the limb still contribute to the total radiation.

Fig. 4 represents for an interval of small solar activity and without gaps in the coronal observations (*cf.* Fig. 2, Plate 14) the total intensity $K$ of the corona, its intensity on the central meridian $K_Z$ and the intensity $\mathcal{J}_{F_2}$ of the ultra-violet radiation

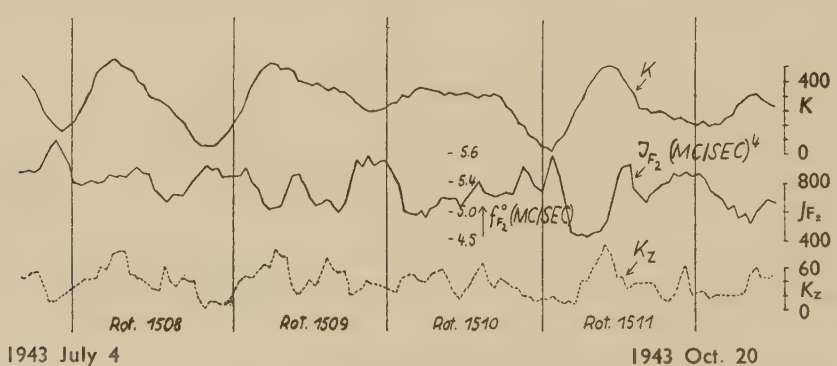


Fig. 4.

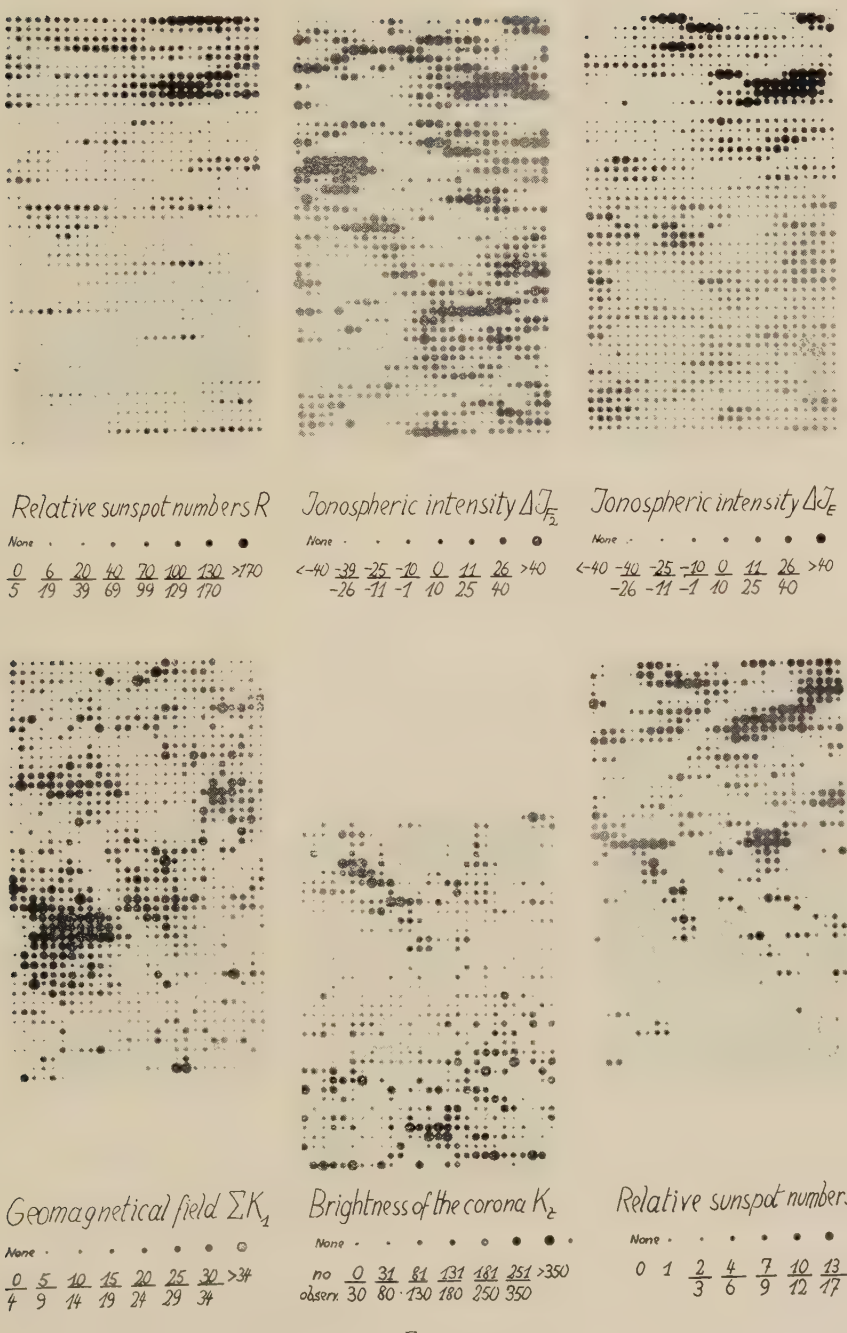
Relative sunspot numbers R
None
0 6 20 40 70 100 130 >170
5 19 39 69 99 129 170
Ionospheric intensity ΔJF2
None
<-40 -39 -25 -10 0 11 26 >40
-26 -11 -1 10 25 40
Ionospheric intensity ΔJE
None
<-40 -40 -25 -10 0 11 26 >40
-26 -11 -1 10 25 40
Geomagnetical field ΣK1
None
0 5 10 15 20 25 30 >34
4 9 14 19 24 29 34
Brightness of the corona K2
None
no observ. 0 31 81 131 181 251 >350
30 80 130 180 250 350
Relative sunspot numbers R2
None
0 1 2 4 7 10 13 >17
3 6 9 12 17

FIG. 2.

K. O. Kiepenheuer, On the Relations Between Ionosphere, Sunspots and Solar Corona.

producing the $F_2$-layer. The total intensity $K$ has been calculated by projecting the limb measurements on the disk, by multiplying them always by the cosine of the distance from the apparent centre of the disk, and by finally adding them up. The negative correlation, particularly between $\mathcal{J}_{F_2}$ and $K$ is striking and will be of importance for the prediction or for the diagnosis of ionospheric conditions or the propagation of short waves with the aid of solar observations. For this reason the corresponding critical frequencies of the $F_2$-layer have been added in Fig. 4. The relation exhibited here between the green corona and the ionosphere comes mainly from the variations of the coronal temperature and not from variations of the electron density, which can be taken from observations of the corona in white light. But the electron density also plays an important part in the emission in the far ultra-violet, making a continuous photometry of the corona in white light very desirable. The measurements made so far without eclipse, due to B. Lyot on the Pic du Midi who uses a polarization method, are not numerous enough to allow quantitative conclusions. From the few measurements available* it follows, however, clearly that the white corona (that means the electron density) shows a positive correlation with the spot numbers and follows these with a greater time lag than the brightness of the green line (i.e. the temperature) does. The supposition, that the slow ultra-violet component II is emitted from these coronal perturbations of electron density, which are controlled by the spots, is therefore quite plausible.

4. *Sunspots and ionosphere.*—The correlation between spot activity and ionospheric ultra-violet intensities discussed in the beginning of this paper and expressed in analytical form by (1) did not make allowance at all for the fact that the spots influence only the radiation processes going on in the higher disturbed levels of the inner corona. It may be asked whether the nature of the relationship between spot and ultra-violet emission can be cleared up by statistical methods.

If we study first, according to the "superposed epoch method", the behaviour of the spot numbers near selected days with exceptionally high or low ionospheric intensities (daily deviations from the rotational mean, component III) we get, as shown in Fig. 5, for all ionospheric layers, a pronounced maximum of spot numbers a few days *before* the selected day. This maximum is the more pronounced the smaller the zone around the apparent centre of the Sun is from which the spot numbers have been taken. In Fig. 5 therefore a central zone with $r = \frac{1}{4} r_\odot$ has been taken. For the $F_2$-layer the maximum comes about 3 days, for the E-layer about 2 days before the selected day. The corresponding curves for selected days with *weak* ionospheric intensities are also plotted and behave as expected nearly in the opposite way apart from the minima being pushed a little more back. Since we cannot interpret this geometrically we have to suppose that the brightness of the rapidly variable ultra-violet radiation sources, e. g. the faculae, follows the development of spot groups, characterized by the number of spots in a group, with a delay of a few days. Exact material about the development of faculae in connection with the state of the corresponding spot is very poor. However, in the author's opinion this delay is in accordance with observations. If, for instance, it is supposed † that the heating of the granulae of faculae is produced by the chromospheric induction currents set up by the varying magnetic field $H$ of a spot group,

* Thanks to a kind communication of Mr B. Lyot in Paris.

† *Ann. Astrophys*, **9**, 42, 1946.

the production of "Joule's heat" will be proportional to $H^2$. If we express the field strength $H$ of a spot by its area—for field strengths which are not too great these are proportional *—we recognize that for a normal development of a spot the greatest spot area, that is the greatest heat production, will be a few days *after* the attainment of the greatest spot number in the group. Therefore we see in the delay of a few days between spot numbers and ultra-violet intensity another argument for the identity of the sources of the rapidly variable ultra-violet component III and the chromospheric faculae.

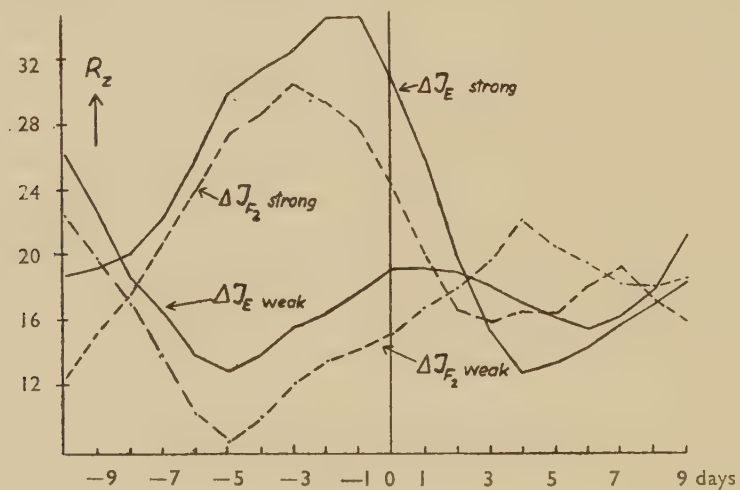


FIG. 5.

The lifetime of such solar ultra-violet sources, as can be seen easily from the 27-day row representation of Fig. 2 (Plate 14), is greater than that of the spots and so fits better the visible lifetime of faculae. Moreover the representation of the ultra-violet radiation in the figure gives some information about their limb darkening. If one counts within a 27-day sequence the number of consecutive days with high ultra-violet intensity, one often counts up to 9 days. This shows evidently that an ultra-violet source on its way of 13 days from the eastern to the western limb of the Sun radiates for about nine days to the Earth. If one considers that the projected area of the faculae (or the area of the disturbed region of the corona) as seen from the Earth varies with the cosine of the distance from the central meridian, the limb darkening of the ultra-violet radiation can be only very small.

Finally, attention may be called to a few conclusions which can be drawn simply by regarding Fig. 2 (Plate 14). This is the pronounced complementary behaviour of the perturbations of the geomagnetic field ($\Sigma K_1$) and the relative spot numbers of a central zone ($R_{ZZ}$), the parallelism of coronal intensities in the central meridian ($K_Z$) and the spot numbers in a central zone ($R_{ZZ}$), the negative correlation between the ultra-violet intensities $\Delta\mathfrak{J}_E$, $\Delta\mathfrak{J}_{F_2}$ (which are not reduced to a constant geomagnetic field) and the perturbations of the geomagnetic field ($\Sigma K_1$), and finally the distinct parallelism between the ultra-violet intensities $\Delta\mathfrak{J}_E$ and $\Delta\mathfrak{J}_{F_2}$.

6. *General conclusions.*—This interpretation of the observed relations between ionosphere and Sun supposes implicitly the existence of a certain rise of temperature or deviation from thermodynamical equilibrium in the outer parts of the

* S. B. Nicholson, *P.A.S.P.*, **45**, 51, 1933.

Sun's atmosphere. To round off the general picture we finally discuss in a few words the primary causes of the heating effects in the chromosphere and in the corona.

As already mentioned the chromospheric faculae, which probably represent the main sources of the rapidly varying ultra-violet component III can be understood quite well as the effect of a system of induction currents over sunspots. In the corona near a spot-group the effects of a varying magnetic field are quite different, as a consequence of the much greater free path of the electrons. The free electrons with kinetic energy $E$, revolving in the varying magnetic field $H$, are accelerated * as in a betatron according to

$$\frac{dE}{dt} = \frac{E}{H} \cdot \frac{dH}{dt}, \tag{6}$$

and may gain between one collision and the next a few hundred volts, enough for ionizing, e. g. *Fe* atoms 15 times. It is a fact, that the coronal lines, almost without exception, are most prominent in the vicinity of spots or faculae. We conclude, therefore, that the magnetic fields of spots are the energy sources for their excitation.

There remains the question, which energy heats the *permanent* corona in spotless regions or during the minimum of solar activity? Knowing that the total intensity of the corona varies very little or not at all during a cycle †, we incline to the proposition of Biermann ‡, Vand §, ten Bruggencate || and others, that the energy comes from the chromospheric turbulence or perhaps partially from the evaporated meteoric matter falling into the Sun. The amount of energy available as well as the energy of the particles should be sufficient to cover the radiation losses of the corona by free-free and free-bound transitions. We conclude, that the ionospheric ultra-violet component I is largely of meteoric origin. The idea that its source is in the corona is strengthened by the fact that this component decreases with increasing wave-length relative to the variable components (*cf.* (1)). This is according to Fig. 1 only possible for coronal temperatures.

The same is true for the slowly variable component II which, similar to I, seems to decrease with increasing wave-length. It is also of coronal origin and comes probably from the regions with varying structure (electron density and temperature) under the influence of sunspots.

The rapidly varying component III which increases, evidently with increasing wave-length, therefore must come from regions with lower temperature than the corona. Also in this way we come back to the faculae.

As has been shown already, the correlation of component II with spot numbers is positive, but the elevation of coronal temperature brings in a negative correlation. We conclude, therefore, that the increase of electron density above and near a spot group is the decisive factor. Indeed, the regions with increased electron density, that is regions where the white corona is brighter, are much more extended than the green or red coronal patches, which also are coupled with spot groups.

* *Ann. Astrophys.*, **8**, 236, 1945.
† M. Waldmeier, *Ergebnisse der Sonnenforschung*, p. 236, Leipzig, 1941.
‡ L. Biermann, *Naturwissenschaften*, **33**, 118, 1946.
§ V. Vand, *Nature*, **151**, 728, 1943.
|| P. ten Bruggencate, *Veröff Univ Sternw. Göttingen*, No. 83, 1947.

The slowness with which the slowly variable component II follows the spot numbers therefore comes from the slow diffusion of coronal electrons.

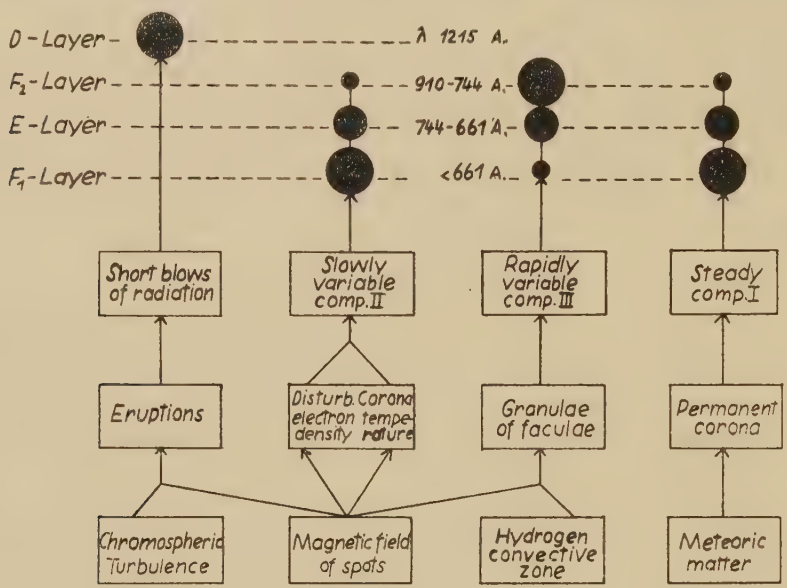


FIG. 6.

We summarize our results in Fig. 6, which also contains chromospheric eruptions. These seem to us to be an extreme kind of granulae of faculae* produced by the combined action of turbulent motion and magnetic field of a spot.

I have to express my thanks to many whose support was indispensable for the collection of the observational material used here. My particular thanks are due to the two indefatigable corona observers on the Wendelstein and on the Zugspitze, Messrs. H. Haffner and A. Bruzek.

*Fraunhofer Institut,*
*Freiburg i. B., Germany :*
1946 *December* 12.

* *Z. Astrophys.*, **20**, 332, 1941.

# NOTE ON STELLAR STRUCTURE

*F. Hoyle and R. A. Lyttleton*

(Received 1946 April 24)

*Summary*

The conditions to be satisfied at a boundary separating convective and radiative regions in a star are considered in detail for the case when radiation pressure is of comparable importance to gas pressure. The effective polytropic index, $n$, appropriately defined, is proved to be continuous if the composition is continuous, but discontinuous if the composition changes abruptly at the interface. Expressions are obtained of the physical conditions that must be satisfied, and it is shown that, if $X$ and $Y$ are the mass proportions of hydrogen and helium, the boundary condition can be stated in the form that $\mu(1-X-Y)(1+X)(n+1)$ is continuous.

1. *Introduction.*—In the mathematical theory of stellar constitution it is sometimes convenient to introduce a quantity $n$, termed the effective polytropic index, usually * defined by the relation

$$1+\frac{1}{n}=\frac{d\log P}{d\log \rho}, \tag{1}$$

where $P$ is the total pressure and $\rho$ the density. More recently, however, a number of writers † have preferred to define the effective index by the relation

$$1+n=\frac{d\log P}{d\log T}, \tag{2}$$

where $T$ is the temperature. When radiation pressure is negligible the two definitions are equivalent, but not otherwise. When radiation pressure is not negligible, the second form of the relation is a more convenient definition, in so far as the introduction of such a quantity may be necessary at all, and for the discussion of this paper the form (2) is adopted.

When the model taken for the star is an Emden polytrope, $n$ is constant throughout. Although complete polytropic models are no longer of much utility (except perhaps in the theory of white dwarfs), the variation of the index $n$ nevertheless provides a useful rough indication of the properties of more complicated models. For example, in the Cowling model, which refers to the case when radiation pressure is negligible, $n$ is 1·5 in the convective core, whilst in the radiative envelope it starts at 1·5 at the surface of the core and gradually increases until a value of 3·25 is reached at the surface of the star. If the integrations for such a model are regarded as computed inwards, the inner boundary is considered to be reached when $n$ has fallen to 1·5. Thus it comes about that the condition of fit between adjoining convective and radiative regions may be conveniently stated in the form that $n$ is continuous across the interface. It is important to realize, however, that the continuity of $n$ is by no means the direct mathematical expression of the thermodynamic condition that must be satisfied at the interface. Indeed, as will be seen

* Sir A. S. Eddington, *M.N.*, **99**, 4, 1938.

† L. R. Henrich, *Ap. J.*, **96**, 108, 1942 and M. Schönberg and S. Chandrasekhar, *Ap. J.*, **96** 162, 1942.

in Section 3, cases of special importance occur in which $n$ is *not* continuous. The position happens to be that $n$ is continuous in some cases, the Cowling model for instance, but not in others. In order to decide the condition to be applied in any given case, the safest procedure is to carry out the thermodynamical investigation appropriate to the special circumstances of the problem. The object of the present paper is to discuss the mathematical expression of the physical conditions that have to be satisfied at the interface between a spherical inner convective region and a surrounding concentric radiative envelope for the following cases:—

(i) When there is no change of composition between the two regions.

(ii) When there is a discontinuous change of composition across the interface.

These two cases are examined in Sections 2 and 3 respectively. Throughout the discussion radiation pressure will be regarded as of comparable importance with gas pressure.

2. *Convective and Radiative Equilibrium when the Composition is Uniform.*—The limiting condition for convection to occur has recently been shown * to be expressible in the form

$$\frac{d}{dr}[T^3/\rho + (3\mathfrak{R}/4a\mu)\log T^{n_0}/\rho] = 0, \tag{3}$$

where $r$ is the central distance in the star, $\mu$ is the mean molecular weight, $n_0 = (\gamma - 1)^{-1}$, where $\gamma$ is the ordinary ratio of specific heats of the material, $\mathfrak{R}$ is the gas constant, and $a$ is Stefan's constant. Denoting the expression in square brackets in (3) by $f$, the condition for radiative equilibrium to be stable is

$$df/dr < 0. \tag{3'}$$

On the other hand only a very slight increase of $df/dr$ above zero produces violent convection, so that convective equilibrium in a star is always attained before $df/dr$ has risen appreciably above zero. For this reason it is usually sufficiently accurate to put $df/dr = 0$ in any region of a star in which the equilibrium is convective.

Consider now an inner convective region of radius $r_c$ surrounded by a radiative envelope. Then

$$df/dr = 0 \text{ for } r < r_c$$

and

$$df/dr < 0 \text{ for } r > r_c.$$

From these it follows that

$$\left(\frac{df}{dr}\right)_{r \to r_c -} \geqslant \left(\frac{df}{dr}\right)_{r \to r_c +}, \tag{4}$$

where $r \to r_c -$ means that $r$ tends to $r_c$ from below, and $r \to r_c +$ means that $r$ tends to $r_c$ from above. It is particularly to be noticed that at the present stage of the argument it is not possible to assert the equality of the two expressions in (4). From physical considerations the following five equalities are seen to hold:

$$\begin{aligned} \mu_{r \to r_c -} &= \mu_{r \to r_c +} \\ P_{r \to r_c -} &= P_{r \to r_c +} \\ T_{r \to r_c -} &= T_{r \to r_c +} \\ M(r)_{r \to r_c -} &= M(r)_{r \to r_c +} \\ L(r)_{r \to r_c -} &= L(r)_{r \to r_c +}. \end{aligned} \tag{5}$$

* R. A. Lyttleton, *M.N.*, **106**, 228, 1946.

The first of these relations arises since the composition is assumed uniform; the second from the requirement that there can be no pressure discontinuity in an equilibrium structure; the third since there can be no temperature discontinuity; the fourth is the condition that the mass $M(r)$ contained in a sphere of radius $r$ must be continuous; and the fifth expresses the condition that the outward flux of energy must be continuous.

The last condition arises from the requirement that there can be no permanent accumulation or deficit of heat at the interface, and though apparently trivial this turns out to have important consequences. Since the pressure $P$ is given by

$$P=\frac{\mathfrak{R}}{\mu}\rho T+\tfrac{1}{3}aT^4, \tag{6}$$

the continuity of $P$, $\mu$ and $T$ evidently requires

$$\rho_{r\to r_c-}=\rho_{r\to r_c+}. \tag{7}$$

Equations (5) and (7) give the continuity conditions satisfied at the interface by the physical quantities concerned. It remains to consider what conditions apply to the derivatives of $P$ and $T$. The derivative of the pressure satisfies the equation

$$\frac{dP}{dr}=-\frac{G\rho M(r)}{r^2} \tag{8}$$

throughout a star in equilibrium, where $G$ is the constant of gravitation. Since all the factors on the right-hand side in (8) are continuous at the interface, it follows at once that

$$\left(\frac{dP}{dr}\right)_{r\to r_c-}=\left(\frac{dP}{dr}\right)_{r\to r_c+}. \tag{9}$$

The condition satisfied by the temperature gradient can be derived from the last relation in (5). The flux of energy can always be written as the sum of two terms, thus

$$L(r)=L^{\mathrm{rad}}(r)+L^{\mathrm{con}}(r), \tag{10}$$

where $L^{\mathrm{rad}}(r)$ is the energy flux carried by radiation at central distance $r$, and $L^{\mathrm{con}}(r)$ is the flux due to convection. Now in the radiative region $L^{\mathrm{con}}(r)=0$ everywhere, while in the convective region both $L^{\mathrm{rad}}(r)$ and $L^{\mathrm{con}}(r)$ are in general different from zero. By the assumptions of the problem, the energy flux due to convection across the boundary of the convective core must be negligible, since otherwise the boundary would not be the limit of the convective zone. Thus it follows that

$$L^{\mathrm{con}}_{r\to r_c-}=0,$$

and this requires

$$L^{\mathrm{rad}}_{r\to r_c-}=L^{\mathrm{rad}}_{r\to r_c+}. \tag{11}$$

Now $L^{\mathrm{rad}}$ is given by the radiative equation whether the equilibrium is radiative or convective, and with Kramers' law for the opacity this equation takes the well-known form

$$\frac{4}{3}acT^3\frac{dT}{dr}=-\kappa(1-X-Y)(1+X)\frac{\rho^2}{T^{3\cdot5}}\cdot\frac{L^{\mathrm{rad}}(r)}{4\pi r^2}, \tag{12}$$

where $c$ is the velocity of light, $\kappa$ is the constant entering from the opacity law, $X$ is the proportion by mass of hydrogen and $Y$ the proportion by mass of helium. (It may be noted that the conclusions that will be reached in this section by using

the foregoing equation would apply equally well if the opacity were modified to include the guillotine factor, or if the opacity were wholly or partially by electron scattering.) Since all the quantities in (12) except $dT/dr$ have already been shown to be continuous at the interface, it follows immediately that this remaining factor must also be continuous; that is

$$\left(\frac{dT}{dr}\right)_{r\to r_c-} = \left(\frac{dT}{dr}\right)_{r\to r_c+}. \tag{13}$$

The relations (5), (6), (9) and (13) taken together are seen to establish

(*a*) the continuity of the polytropic index as defined by (2),

(*b*) the continuity of $df/dr$,

when there is no change of composition across the interface.

3. *Change of Composition across the Interface.*—The special case will be considered in which $\mu$ is constant in the convective region and also constant but with a different value in the radiative region; that is, $\mu$ changes discontinuously at the interface between the convective core and the radiative envelope. (It is not the purpose of the present paper to consider to what extent such a discontinuous change may be approached physically, but simply to derive on the basis of established theory the conditions that must be satisfied if a discontinuous change in $\mu$ is assumed.) The important result will be obtained that the index $n$ is no longer continuous at the interface.

The first of conditions (5) no longer holds, and as a result (7) is replaced by

$$\left(\frac{\rho}{\mu}\right)_{r\to r_c-} = \left(\frac{\rho}{\mu}\right)_{r\to r_c+}. \tag{7'}$$

Equation (8) remains valid, but since $\rho$ is not continuous (9) has now to be replaced by

$$\left(\frac{1}{\rho}\frac{dP}{dr}\right)_{r\to r_c-} = \left(\frac{1}{\rho}\frac{dP}{dr}\right)_{r\to r_c+}. \tag{9'}$$

Equations (11) and (12) also remain valid, but (13) is modified to

$$\left[\rho^{-2}(1-X-Y)^{-1}(1+X)^{-1}\frac{dT}{dr}\right]_{r\to r_c-} = \left[\rho^{-2}(1-X-Y)^{-1}(1+X)^{-1}\frac{dT}{dr}\right]_{r\to r_c+}. \tag{13'}$$

This condition can be put in a form involving $n$ by making use of the already established continuity of $P$, $T$ and $1/\rho \cdot dP/dr$; thus the expression

$$\rho^{-2}(1-X-Y)^{-1}(1+X)^{-1}\frac{1}{T}\frac{dT}{dr} \div \left(\frac{1}{\rho P}\frac{dP}{dr}\right) \tag{14}$$

must be continuous across $r = r_c$. The ratio of logarithmic differential coefficients occurring here can now be expressed in terms of $n$ by means of (2), and hence the continuity of (14) requires that

$$\rho^{-1}(1-X-Y)^{-1}(1+X)^{-1}(n+1)^{-1}$$

shall be continuous. Finally, by means of (7'), this condition becomes that

$$\mu(1-X-Y)(1+X)(n+1) \tag{15}$$

must be continuous across the interface.

Thus, if $\mu$ changes abruptly at the interface, (15) shows that $n$ must do so too, and accordingly the assumption in this case that $n$ is continuous is seen to be

erroneous. The argument also shows that $df/dr$ is discontinuous in this case. If helium is assumed to be absent or unimportant, so that $Y=0$, the continuity of (15) reduces to the condition derived in a previous paper *, namely $\mu(1-X^2)(n+1)$ continuous.

The preceding discussion, it may be noticed, would apply unaltered to the case where the regions of different molecular weight are both in radiative equilibrium. Accordingly, if a change of composition occurs across an interface on both sides of which equilibrium is radiative, condition (15) will again give the appropriate relation between the values of $n$ on the two sides. It will also be necessary that condition (3′) shall be satisfied on both sides of the interface in order that the assumed radiative equilibrium shall be stable.

4. The foregoing detailed derivation of the continuity conditions between convective and radiative regions has been prompted by a number of recent papers by Chandrasekhar and his collaborators †, in which the continuity of the effective polytropic index appears to be regarded as in itself the direct expression of a physical condition. For the most part these papers have been concerned with cases in which $\mu$ is uniform, and for the reasons explained above the assumption of continuous $n$ has not led to error in conclusions based on it. But in a further investigation of stellar evolution ‡, in which non-uniform composition is introduced, the assumption of continuous $n$ has led to the adoption of a stellar model that in reality is not a physically consistent and permissible one, and the arguments based on this model must accordingly stand in need of reconsideration.

In the investigation in question it is assumed that helium plays no important part, that is $Y=0$. (This in itself, while permissible mathematically, is an extremely dubious assumption from a physical standpoint in the particular problem of stellar evolution examined by these authors §, since the very process of exhausting the hydrogen of the core of a star is likely to bring about a considerable helium content.) It is also assumed that radiation pressure is negligible and that a decrease of molecular weight by a factor one-half occurs across the interface, the convective core being supposed of molecular weight 2 and the radiative envelope of molecular weight 1. Denoting the regions by suffixes 1 and 2 respectively, so that $\mu_1=2\mu_2$, and applying (15), the proper relation to be satisfied for these assumed conditions is that

$$\mu_1(1-X_1^2)(n_1+1)=\mu_2(1-X_2^2)(n_2+1). \qquad (16)$$

Adopting the simple form $\mu=2/(1+3X)$, corresponding to the case $Y=0$, the appropriate values are

$$\mu_1=2, \qquad X_1=0; \qquad \mu_2=1, \qquad X_2=\tfrac{1}{3};$$

together with

$$n_1=1{\cdot}5.$$

Hence at the inner boundary of the radiative region the correct value of the effective index in such a model would be

$$n_2=4{\cdot}625.$$

But it has been shown by the writers in an earlier paper || that, when radiation

* *M.N.*, **102**, 220, equation (7), 1942.
† *Ap. J.*, loc. cit.,
‡ M. Schönberg and S. Chandrasekhar, *Ap. J.*, **96**, 162,1942.
§ M. Schonberg and S. Chandrasekhar, *loc. cit.*
|| F. Hoyle and R. A. Lyttleton, *M.N.*, **102**, 177, 1942.

pressure is negligible, if $n$ once rises above the value 3·25 at any interior point of a radiative region, it must continue to increase with $r$. There can therefore be no possibility in the present instance of $n$ behaving in such a way that it eventually tends to 3·25 from below as the surface is approached, which is an essential property of an equilibrium model. Thus a configuration of the type considered by Schönberg and Chandrasekhar, in which $\mu$ changes abruptly by a factor of two, is not a physically possible model.* That a seemingly permissible mathematical solution is nevertheless found is to be attributed to the assumption of the continuity of $n$ as the changeover condition, and it is clear from the present discussion that an examination of the model so obtained must show that in reality it cannot satisfy the necessary physical requirements at both the interface and the free surface.

*St. John's College,*
*Cambridge:*
1946 *April* 22.

* The limiting values of the factor by which $\mu$ can change in equilibrium models have been determined by the writers, for the case when $Y=0$, in *M.N.*, **102**, 220, 1942.

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# MONTHLY NOTICES

OF THE

# ROYAL ASTRONOMICAL SOCIET

Vol. 106 No. 1 1946

## ANNUAL REPORT OF THE COUNCIL

*Published and Sold by the*

ROYAL ASTRONOMICAL SOCIETY
BURLINGTON HOUSE
LONDON, W. 1

*Price Six Shillings*

## NOTICE TO AUTHORS

1. *Communications.*—Papers must be communicated to the Society by a Fellow. They should be accompanied by a summary at the *beginning* of the paper. Authors are requested to submit MSS. in duplicate, when practicable.

2. *Presentation.*—Authors are allowed considerable latitude, but they are requested to follow the general style and arrangement of *Monthly Notices*. References to literature should be given in the standard form, including a date, for printing as footnotes.

3. *Notation.*—Authors should conform closely with the recommendations of Commission 3 of the International Astronomical Union (*Trans. I.A.U.*, Vol. VI, p. 345, 1938). Council has decided to adopt the I.A.U. 4-letter abbreviations for constellations where contraction is desirable (Vol. IV, p. 221, 1932).

4. *Diagrams.*—These should be drawn about twice the size required in print and prepared for direct photographic reproduction except for the lettering which should be inserted in pencil. Legends should be given in the manuscript indicating where in the text the figure should appear. Photographs are normally reproduced true to scale.

5. *Tables.*—These should be arranged so that they can be printed upright on the page.

6. *Proofs.*—Costs of alteration exceeding 5 per cent. of composition must be borne by the author. Fellows are warned that such costs have risen sharply in recent years, and it is in their own and the Society's interests to seek the maximum conciseness and simplification of symbols and equations consistent with clarity.

7. *Revised Manuscripts.*—When papers are submitted in revised form it is especially requested that they be accompanied by the original MS.

### *Reading of Papers at Meetings*

8. If a paper is to be read at a meeting it should be received by the Assistant Secretary not later than the previous Saturday.

9. When submitting papers authors are requested to indicate whether they will be willing and able to read the paper at the next or some subsequent meeting, and approximately how long they would like to be allotted for speaking.

10. The general issue of postcards giving the programme of each meeting has been temporarily discontinued, and has been replaced by the limited circulation of abstracts of papers to be read. Announcements of organised discussions, however, will be made in advance when practicable.

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*Printed in Edinburgh by Neill & Co., Ltd.*, 1946